VŠB — TECHNICAL UNIVERSITY OF OSTRAVA
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Optimalizace portfolia v rámci mean-variance přístupu
Portfolio Optimization under Mean-Variance Framework

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Description:

1. Introduction
2. Description of Investment Decision-Making
3. Description of Portfolio Optimization Problem
4. Application of Portfolio Optimization Problem
5. Conclusion

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References:

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The declaration

"Herewith I declare that I elaborated the entire thesis, including all annexes, independently."

04/25/2015

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Contents

1. Introduction

In a modern society, almost everyone owns a portfolio of assets. A good portfolio is more than a long list of goods stocks and bonds. It is a balance whole, proving the investor with protections and opportunities with respect to a wide range of contingencies *(Markowitz, 1971)*. In general, the portfolio is likely to contain real assets, such as a car, a house, as well as financial assets. The composition of the portfolio may be the result of series of haphazard and unrelated decisions, or it may be the result of deliberate planning. An investor is faced with a choice from among an enormous number of assets *(Elton et al. 2009)*. Thus, it is important for investors to know the assets allocation construction, due to portfolio optimization is closed related to investment decision-making and consequences of investment.

Portfolio optimization is a formal mathematical approach to making investment decisions across a collection of financial instruments or assets. The classical approach, known as modern portfolio theory (MPT) was set by Markowitz in 1952. The portfolio optimization problem is unending research focus of both academics and practitioners. The objective of the thesis is to perform ex-post analysis of portfolio different optimization problems, which arise from different risk attitudes.

This thesis is divided into five chapters. The first chapter provides the introductions which expound the background material and structure of the thesis.

In the second chapter, the description of investment decision-making can be found. There are four sections which are analysis about investors. According to three different types of risk attitude, the different utilities of investors are explained in first section; in the second section, the financial position and the objective of investors are exposed; then we have information about environment of investment which is introduced financial instruments. Given this background, finally we can made decision of investment.

Financial modeling in very simple terms is systematic creation of a logical structure, to process and analyze a data set, so as to arrive at conclusive financial decisions which can represent the performance of a financial asset or portfolio of a business, project, or any other investment. Chapter 3 develops description of portfolio optimization problem by using two types of financial models, which can be utilized to analyze the efficient set and optimal portfolio. They are: Markowitz model, in which focus on finding of the efficient frontier, is to set up the efficient Markowitz set; The Black's model is the second one, which is similar to Markowitz model, but the constraints are a little different. Besides, we also can find the theory of mean-variance framework which is a great contribution to economics by Markowitz. We analysis the dataset which from yahoo finance according to this theory and two types of financial model; on the other hand, the strategies of model analysis and performance measures can be found in this chapter. This is theoretical part, all formulas and definitions relevant to the application part are defined here.

Based on Chapter 3 description, application of portfolio optimization problem is presented in Chapter 4. In this chapter, we can find all the procedure of problem solution step by step, both in Markowitz and Black's model. The results of historical data are clearly presented and all results and interpretation of comparison between two models are commented in sufficient details. Finally, there are discussions, analysis, and recommendations of all problems, such as compared Sharpe ratios to portfolio optimization problems between different models, which show how to determine the optimal investment. The objective of this chapter is assessing financial model to help investors to make their investment decision.

The last chapter is conclusion. In this chapter the findings about the portfolio optimization under mean-variance framework and results obtained in the thesis are summarize.

2. Description of Investment Decision-Making

An investment is the current commitment of money or other resources in the expectation of reaping future benefits.[1] In this chapter, the fundamental analyses of investment decision-making are described. There are four main sections which are description of investors contain risk attitude, financial position, objective of investment and decision-making of investment.

The investor can be the trader, fund manager, director or investment manager. They made decisions as to how, when, where and how much capital will be spent on investment opportunities and the aim of the investors is to seek the maximize returns while minimizing risk. in an asset allocation problem. The investment decisions are supported by decision tools and follow research to determine costs and returns for each option. In order to determine the optimum allocation, the investor needs to model, estimate, access, and manage uncertainty. The most popular approach to asset allocation is the mean-variance framework.

2.1 Risk attitude of investors

Investing has a major role in financial planning due to our reliance on investments to increase our wealth and assist us in reaching our financial goals.

Despite many people are hesitant about investing as they are lack of knowledge of investment, thus they would like to find portfolio managers or investment intermediary to help them. The portfolio theory is often applied to help the investor achieve a satisfactory return compared to the risk undertaken.

A classic example of rational investment decision in finance, the fact revealed the importance of risk in investment process and describe how risk − willing investors. There are three types of risk attitude: risk-averse, risk-neutral and risk-seeking.

a) Risk-averse: is a concept in economics and finance, based on the behavior of humans (especially consumers and investors) while exposed to uncertainty to attempt to reduce that uncertainty. (e.g. If there are two investments with similar expected return, the

[1] *BODIE et al. (2010)*

investor who prefers the investment with lower risk.) It is illustrated in Chart 2.1:

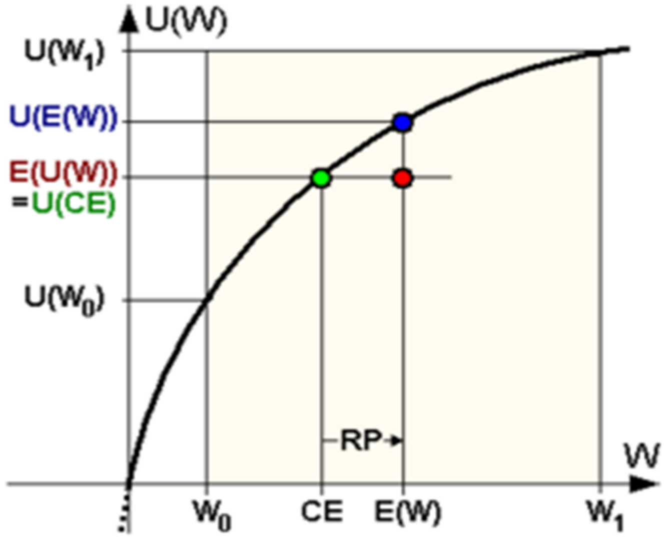

Chart 2.1: Utility function of a risk-averse investor
Source: http://en.wikipedia.org/wiki/Risk_aversion

where, *CE* is certainty equivalent, $E(U(W))$ is expected value of the utility (expected utility) of the uncertain payment, $E(W)$ is expected value of the uncertain payment, $U(CE)$ is utility of the certainty equivalent, $U(E(W))$ is utility of the expected value of the uncertain payment, $U(W_0)$ is utility of the minimal payment, $U(W_1)$ is utility of the maximal payment, W_0 is minimal payment, W_1 is maximal payment and *RP* is risk premium.

The concave shape of the curve reflects the assumed diminishing marginal utility of this wealth. It is assumed diminishing marginal utility of income that gives rise to the risk aversion. Chart 2.1 illustrates risk aversion. There are assumed three options. To examine the person's preferences among these options, we must compute the expected utility available from each. a) Retain the current level of wealth without any risk; b) Take a fair bet with a chance of winning or losing some money.

b) Risk-neutral: The investors who have no sensitivity to risk and just try to find the maximum expected return. There is some difference linear trend which is illustrated in Chart 2.2.

According to Chart 2.2, we can find the expected value of the utility, utility of the expected value of the uncertain payment and utility of the certainty equivalent cross in one point, and there is no difference between expected value of the uncertain payment and certainty equivalent which means no risk premium here. So the risk-neutral can't influence the utility of investors.

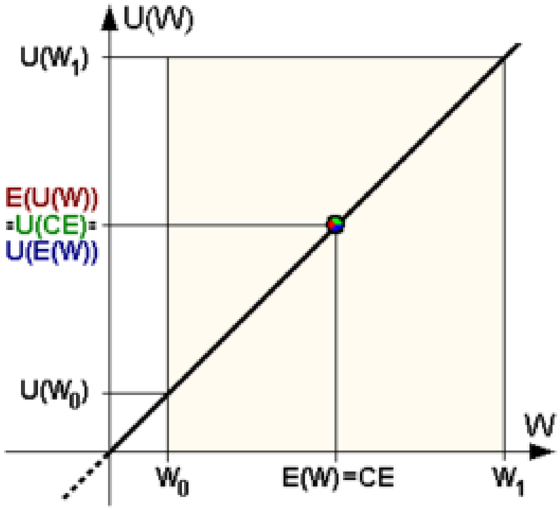

Chart 2.2: Utility function of a risk-neutral investor
Source: http://en.wikipedia.org/wiki/Risk-neutral

c) Risk-seeking: The investor who prefers to take big risk to increase the potential return on investment. It is illustrated in Chart 2.3.

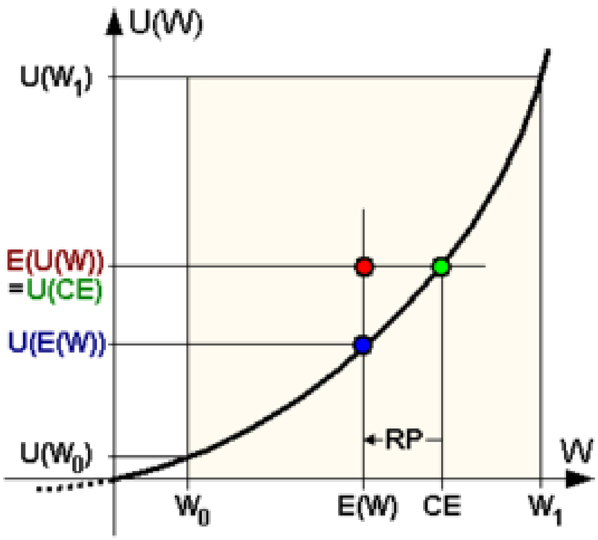

Chart 2.3: Utility function of a risk-seeking investor
Source: http://en.wikipedia.org/wiki/Risk-seeking

It is convex shape which utility of the certainty equivalent is higher than utility of the expected value and certainty equivalent is bigger than expected value of the uncertain payment. So there is risk premium.

Diminishing marginal utility of income means that people will be averse to risk; among options with the same expected value, people will prefer risk free income to risky option; In fact, a person would be willing to give up some amount of income to avoid taking a risk (risk premium).

Table 2.1: Summary of Investment Needs by Client Type[2]

Client	Time Horizon	Risk Tolerance	Income Needs	Liquidity Needs
Individual investors	Varies by individual	Varies by individual	Varies by individual	Varies by individual
Defined benefit pension plans	Typically long-term	Typically quite high	High for mature funds; low for growing funds	Typically quite low
Endowments and foundations	Very long-term	Typically high	Sufficient to meet spending commitments	Typically quite low
Banks	Short-term	Quite low	Sufficient to pay interest on deposits and operational expenses	High to meet repayment of deposits
Insurance companies	Short-term for property and casualty; long-term for life insurance companies	Typically quite low	Typically low	High to meet claims
Investment companies	Varies by fund	Varies by fund	Varies by fund	High to meet redemptions

In the thesis, we assume that all investors are risk-averse.

2.2 Financial position and investment objective of investors

In the beginning of the investing cycle, it should start by determining how much can and need to invest. The investor need to build a budget and analyze their own cash flows to

[2] *Mc MILLAN et al. (2011)*

calculate how much they can afford to invest. The budget will assist in determining amount of investment which is the income they have left over after expenses.

According to the survey of investment objective (may come in the form of a questionnaire), investment advisors and other asset managers are aim in determining the optimal portfolio mix for the client.

There are some examples of investment objectives:

- **short-term goal:**
    a) children's education;
    b) saving for a major purchase;
    c) starting a business;
- **long-term goal:**
    a) retirement.

The investors can create a simple estimate on how much you will need to retire and make a list of your other future expenses. It's wise to determine how much capital you will need in the future and do the investment early.

2.3 Financial instruments

The types and amounts of the assets should be based on the asset allocation strategy. We can find classification of investment assets and financial instrument in this part. Once the investors have determined the right asset allocation strategy for them, then choose individual bonds, equities, mutual funds and other financial instruments for portfolio. Speaking of financial instruments, you can find a variety of them in different market. We can distinguish financial markets into three. There are money market (short-term), capital market (long-term) and derivation market.

The money market

Money market has instruments with short terms to maturity (less than 1 year) with the least price fluctuations and the least risky instruments. The main money market instruments are as follows:

a) Treasury bills are issued by government for up to 6 months maturity which is the most liquid instrument in the money market with no risk associated. (Refer in particular to USA T-bill)

b) Bank certificate of deposit (CD) pay interest at certain times and pay back the full certificate amount at maturity which are issued by commercial banks, corporations, mutual funds, government agencies, etc. It is considered a secure investment for a range from 3 to 5 years.

c) Commercial papers is unsecured, short-term debt instruments which are issued by a corporation, typically for the financing of accounts receivable, inventories and meeting short-term liabilities. Maturities on commercial paper rarely range any longer than 270 days.

The capital market

Capital market has debt and equity instruments with maturities of greater than 1 year; they have far wide price fluctuations and are considered to be risky investments. The main instruments in capital market are as follows:

a) Stocks are an equity claims on the net income and assets of a corporation. Stocks have the following advantages: relatively low commission costs; easy to buy and sell; on the spot priced; can gain dividends; and has potential capital gain. It has a proven track record of being rewarding investment over time. It is considered a risky investment with unlimited profit potential.

b) Corporate bonds are long term bonds which are issued by corporations with strong credit ratings, interest paid once/twice a year and face value upon maturity.

c) Government securities are the most liquid security traded in the capital market.

d) Government agency securities are long term bond which are issued by various government agencies.

e) State and local government bonds are long term debt instrument which are issued by state and local government agencies and tax exempted.

Derivative market

Derivative markets are investment markets that are geared toward the buying and selling of a certain type of securities, or financial instruments. The following are the most often traded types of derivatives markets:

a) Forward contract is the contract that one party agrees to buy and the counterparty to sell a physical or financial asset at a specific price on a specific date in the future.
b) Futures contract is a forward contract that is standardized and exchange-traded.
c) Swaps are agreements to exchange a series of payments on periodic settlement dates over a certain time period (e.g. quarterly payments over two years).
d) Options contract gives its owner the right, but not the obligation, to either buy or sell an underlying asset at a given price (the exercise price or strike price).

2.4 Investment decision-making

The investment decision making process and the investment behavior of individuals and institutions can be an important source of investment success. We can explain that as individuals and institutions, they are looking after an investment opportunity which surplus of cash for some time in the future and they are satisfied with the highest return during the investment period with reasonable associated risk on the expected return after maturity. In fact, we believe that superior decision making may be a much more powerful asset than an information advantage in equity investing, particularly with the advent of electronic dissemination of information and regulations intended to control differential access to information.[3]

Before we made decision of investment, there are few important critical criteria which should be considered: a) understanding (always research and understand what you are investing in.); b) timing (timing the transactions can make a portfolio and carefully plan to sell or buy securities.); c) attention (pay attention to global economics and the business cycle). And we also need to think about which factors affect the investment decision and hence the

[3] *BERNSTEIN et al. (1998)*

selected investment instruments. The impact factors of investment instruments are as follows:

1. Amount of investment (wealth) - The total resources owned by the investor. The increase in wealth raises the quantity demand on an asset.
2. Expected return from the investment - The expected return from the investment over the next period. An increase in an asset's expected return relative to that of an alternative asset raises the quantity demanded for that asset.
3. Investment risk - Investment risk occurs when the expected investment return is not realized. On the one hand, it is common that the higher the risk associated with the investment the higher the return that can be realized. (e.g. stock market where there is unlimited level of profit); and on the other side, in the event of the exchange goes down during financial crisis like the one that happened which led to a decline of some case index by more than 50%, there is a possibility of losing the invested money and this serious situation led to huge capital losses to the investors. Risk adverse investor will go after the investment with lower risk investment like treasury bills. Money market instruments have the no risk since the banks bears all the risk of the investment. A diversified portfolio of assets can eliminate the non-systematic risk and optimal total investment portfolio to the value of systematic risk. If the risk of assets rises, the quantity demand of the assets will go down when the investors hold everything constant.
4. Liquidity - The speed and ease of transferring assets into cash in a short time. If the assets with high liquidity, they can be easily bought or sold. It is safer to invest in liquid assets because it is easier for an investor to get money out of the investment.
5. Time Value of Money (TVM): This concept refers to the fact that the value of a dollar in hand today is not worth as the same value of a dollar tomorrow.

This concept compares the future return of an investment to the present cost paid today in the investment. The investor is willing to pay if the present value of the cost is less than its present value of the incomes. This concept is used by financial managers and investors to assess the investment opportunity which is very important.

Then, you can determine the optimal asset allocation. Asset allocation is the process of deciding how much money of the investors can be invested into the different categories, such as stocks, bonds and cash. The right asset allocation strategy for the situation is heavily dependent on the amount of the investment capital and the future costs and the risk tolerance of investors should be taken into consideration as well. In the most likely scenario, the investor will have to allocate more resources to high reward investments which are risky, if they are lack of the investment capital. On the other hand, the investors should be able to reduce exposure to risk assets if they have a higher savings rate.

In the thesis, there are some "blue chip stocks" which are from Dow Jones Industrial Index with high return and low risk. The objective of the thesis is help the investor manage their money and optimization the portfolio of equities to get the maximum wealth.

3. Description of Portfolio Optimization Problem

The beginnings of modern portfolio theory back to May 1952 when *Markowitz (1952)* published a paper entitled "Portfolio Selection." In it, he showed how to create a frontier of investment portfolios, such that each of them had the greatest possible expected rate of return, given their level of risk.[4] The fundamental goal of portfolio theory is to optimally allocate investments between different assets. Mean variance optimization is a quantitative tool which will allow making this allocation by considering the trade-off between risk and return.

In this Chapter, there are two main models introduced which are Markowitz and Black's model. A financial model is anything that is used to calculate, forecast or estimate financial numbers, which used as tools to assess the attractiveness of an investment. Certain principles of portfolio theory are fundamental: decision makers are risk-averse; they prefer portfolios with higher return and lower risk. From these principles, optimization models that construct efficient portfolios of assets can be developed.

3.1 Mean-Variance Framework

The mean-variance analysis (with 'mean' used interchangeably with average or expected return, and 'variance' used to denote risk). Markowitz demonstrated that under certain conditions, an investor's portfolio selection can be reduced to balancing two critical dimensions: (1) the expected return of the portfolio, and (2) the risk or variance of the portfolio. In context of a portfolio, the total risk of a security can be divided into two basic components: systematic risk (also known as market risk or common risk), and unsystematic risk (also known as diversifiable risk). Modern Portfolio Theory assumes that these two types of risk are common to all portfolios.

Due to the risk reduction potential of diversification, portfolio investment risk, measured as its variance, depends upon both individual asset return variances as well as the 'covariance'

[4] *MARKOWITZ (1971)*

of pairs of assets. In other words, *Markowitz (1952)* states that portfolio selection should be based on overall risk-reward characteristics, as opposed to simply compiling portfolios with securities with individually attractive risk-reward characteristics. Diversification is, in fact, the core concept of Modern Portfolio Theory and directly relies on the conventional wisdom of “never putting all your eggs in one basket” the mean-variance analysis (with 'mean' used interchangeably with average or expected return, and 'variance' used to denote risk).[5]

Mean-variance analysis is a component of modern portfolio theory, which assumes investors make rational decisions, and that for increased risk they expect a higher return. There are two major factors in mean-variance analysis: variance and expected return. Variance represents risk of portfolio and the expected return is an assessment on the return of portfolio. If two investments have the same expected return, but one has a lower variance, the one with the lower variance is the better choice.

By looking at the expected return and variance of an asset, investors attempt to make more efficient investment choices seeking the lowest variance for a given expected return, or seeking the highest expected return for a given variance level. By combining stocks with different variances and expected returns in a portfolio (diversification), the variance and expected return of the portfolio can be changed as the weights move in the portfolio.

Assume the portfolio composed of N assets. Further, consider that we know the joint probability distribution of assets’ future returns, i.e. we know both the expected returns of particular assets $E(R)=\{E(R_1),\ldots,E(R_N)\}^T$ and the covariance matrix of returns $\mathbf{Q}=\{\sigma_{i,j},i=1,\ldots,N,j=1,\ldots,N\}$. Then, assuming the portfolio composition $x=\{x_1,\ldots,x_N\}^T$, we can compute the portfolio expected return $E(R_p)$ and portfolio variance σ_p^2 (standard deviation σ_p respectively) as follows,

$$E(R_p)=\sum_{i=1}^{N}x_i\cdot E(R_i)=x^T\cdot E(R_i), \tag{3.1}$$

$$\sigma_p^{\ 2}=\sum_{i=1}^{N}\sum_{j=1}^{N}x_i\cdot\sigma_{i,j}\cdot x_j=x^T\cdot Q\cdot x, \tag{3.2}$$

$$\sigma_p=\sqrt{\sigma_p^{\ 2}}. \tag{3.3}$$

[5] *HAUGEN (2000)*

3.2 Optimal Risky Portfolio

Modern Portfolio Theory is a financial theory used to assist investors in creating a portfolio that minimizes the market risk for a given expected return, or maximizes the expected return for a given level of market risk. The portfolio's overall risk is minimized further through diversification within the portfolio's assets. In this part, we collected adjusted closing prices dataset of stocks and acquire the monthly total return data on blue-chip stocks and the Dow Jones Industrial Average (DJIA) from Yahoo! Finance.

A market index is used as the benchmark, since market return is the main driver of asset returns in capital asset pricing. We will implement a portfolio optimization methodology based on capital asset pricing and mean-variance analysis. And many model be used to analysis these data and find out which portfolio is the best for our investing. The details of data collection you will find in chapter 4.1, and the source of the data are introduced as well.

3.2.1 Unconditional Historical Estimation of Stock Parameters

Consider the time series of prices for stocks, such as American Express Company (AXP), The Boeing Company (BA), Caterpillar Inc. (CAT), Cisco Systems, Inc. (CSCO) and Chevron Corporation (CVX) and so on which are within the period of 115 months including the price at beginning of the first month. After data input to the table, first, after selecting these assets and determining their monthly prices, the assets' return is calculated. Asset return is the monthly percentage increase of the price, and can be written as follows:

$$R_{i,t} = \frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}} = \frac{P_{i,t}}{P_{i,t-1}} - 1, \tag{3.4}$$

where $R_{i,t}$ is the return from month $t-1$ to month t of asset i, $P_{i,t}$ is the price at month t of asset i, $P_{i,t-1}$ is the price at month t-1 of asset i.

Then we got the capital stocks returns, next make an average of them:

$$E(R_i) = \frac{1}{T} \cdot \sum_{t=1}^{T} R_{i,t}. \tag{3.5}$$

Thereafter it is necessary to define and distinguish between correlation and covariance. Correlation is the measure of how two assets interact with one another, and it can vary between -1 and 1. A correlation of 1 indicates that the two assets react consistent; a correlation of -1 indicates that the two assets move exactly opposite to one other; and a correlation of 0 indicates that the two assets have no connection whatsoever in market shifts. The effectiveness of diversification depends heavily on the correlation coefficients between pairs of assets. The equation for calculating the correlation coefficient is shown below:

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \cdot \sigma_j} = \frac{\sum_{t=1}^{T}[R_{i,t} - E(R_i)] \cdot [R_{j,t} - E(R_j)]}{\sqrt{\sum_{t=1}^{T}[R_{i,t} - E(R_i)]^2} \cdot \sqrt{\sum_{t=1}^{T}[R_{j,t} - E(R_j)]^2}}, \tag{3.6}$$

where ρ_{ij} is the correlation between asset i and j, R_i is the return of asset i, R_j is the return of asset j, T is the sample size.

Population standard deviation of each capital stock return, it is as follows:

$$\sigma_i = \sqrt{\frac{1}{T} \cdot \sum_{t=1}^{T}[R_{i,t} - E(R_i)]^2}. \tag{3.7}$$

Covariance, much like correlation, is also a measure of the amount by which two assets alter over time. However, its magnitude is different. Covariance can be calculated from the two assets' correlation, as follows:

$$\sigma_{ij} = \rho_{ij}\sigma_i\sigma_j = \frac{1}{T} \cdot \sum_{t=1}^{T}[R_{i,t} - E(R_i)] \cdot [R_{i,t} - E(R_j)], \tag{3.8}$$

where σ_{ij} is the covariance between asset i and j, ρ_{ij} is the correlation between asset i and j, σ_i is the standard deviation of asset i, σ_j is the standard deviation of asset j.

3.2.2 Portfolio of Risky Assets

Portfolios of risky assets are constructing risky portfolios to provide the lowest possible risk for any given level of expected return. Thus, the expected return of the portfolio is needed. It is calculated as the weighted average of expected returns of the individual assets within the portfolio, and can be written as follows:

$$E(R_p) = \sum_{i=1}^{N} w_i \cdot E(R_i), \tag{3.9}$$

where $E(R_p)$ is the expected return of the portfolio, w_i is the fraction of the portfolio invested in asset i, $E(R_i)$ is the expected return of asset i. However, the portfolio fractions w_i are subject to two constraints, namely, $\Sigma w_i = 1$, and $0 \leq w_i \leq 1$ where $i=1,\ldots, N$.

Then the market risk is calculated as the variance of the portfolio's return, which can be written as follows:

$$\sigma_p^{\,2} = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i \cdot \sigma_{ij} \cdot w_j = \sum_{i=1}^{N} w_i^{\,2} \sigma_i^{\,2} + \sum_{i=1}^{N}\sum_{\substack{j=1 \\ j\neq i}}^{N} w_i w_j \sigma_{ij}, \tag{3.10}$$

where σ_p^2 is the variance of the portfolio, w_i is the weight of the funds invested in asset i, σ_i is the standard deviation of asset i, w_j is the fraction of the funds invested in asset j, σ_{ij} is the covariance between asset i and j. The portfolio's return volatility or Standard Deviation (SD) comes from the variance of the portfolio, and is calculated as follows:

$$\sigma_p = \sqrt{\sigma_p^{\,2}} = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} w_i \cdot \sigma_{ij} \cdot w_j}, \tag{3.11}$$

once the model is built, the portfolio variance (σ_p^2) is minimized by manipulating the weights of the assets (w_i where $i = 1,\ldots,N$) in the portfolio. Extra constraints can be added, such keeping the return above a certain value or constraining individual portfolio weights.

3.2.3 Utility function of mean-variance portfolio

A portfolio manager manages a portfolio of stocks. The manager has at disposal the amount of W=1 c.u., which should be invested into assets. Suppose that expected returns, standard deviations and correlation matrix for the given period are known. The task of this part is selecting the optimal mean-variance portfolio for a risk-averse investor. The optimization is based on expected utility criterion U with the coefficient of risk attitude k.

The utility function can be formulated as follows,

$$U = E(R_p) - k \cdot \sigma_p{}^2. \tag{3.12}$$

By the objective function we express the maximization of the expected utility of returns U. Simultaneously, $E(R_p)$ is the mean of portfolio return, k gives the attitude to risk, $\sigma_p{}^2$ is portfolio variance.

3.3 Markowitz model

The financial model of mean-variance analysis, developed by Harry Markowitz in 1952 assumes that investors prefer greater return and lower risk. The model treats any portfolio as a single point in the mean-variance Markowitz claims that it is not enough to consider the characteristics of individual assets when forming a portfolio of financial securities. Investors should take into account the movements represented by covariances of assets.

The basic concept of Markowitz portfolio theory is that the instruments in an investment portfolio are not selected individually and it is important to consider how each instrument changes in price relative to how every other instrument in the portfolio changes in price. Investing is a compromise between risk and expected return. For a given amount of risk, Markowitz model describes how to select a portfolio with the highest possible expected return or, for a given expected return; it explains how to select a portfolio with the lowest possible risk.

The Markowitz model is based on many assumptions. According to *Markowitz (1952)*, the key premises are:

1. Approximately normal distribution of financial instruments' returns and parameters of this distribution can be correctly estimated (on average) by the user of the model.
2. Approximately constant correlations of financial instruments' returns over time and correlations can be correctly estimated (on average) by the user of the model.
3. The user of the model is risk-averse and rational. Financial markets are efficient.
4. The user of the model is a price taker, not a price maker (i.e. the investor is unable to influence the exercise price of his/her investment by any invested amount of money).
5. Financial instruments inserted into portfolios are homogeneous.
6. The user of the model can enter long positions only (i.e. buy financial instruments at the beginning of the investment horizon and sell them at the end), not short positions (i.e. sell them and then buy).
7. By market risk, we consider both positive and negative deviations of the actual return from the expected one.

3.3.1 Setting up the feasible and efficient set

First of all, we can think of all possible portfolios we are able to construct. Assume that an investor can invest only into risky assets and no short selling is allowed. The set of all portfolios, the investor is capital to create, is called feasible set. These portfolios are described by two parameters- expected return and standard deviation; however, they are characterized by the portfolio composition $x=\{x_1,\ldots,x_N\}^T$, where $x_i \geq 0$ (no short selling allowed) and $\sum_1^N w_i = 1$ (we invest the whole amount). It is illustrate as Chart 3.1.

As we know all possible portfolios that we can create, we are interested to know which portfolio is the best (so-called optimal portfolio). The choice of optimal portfolio depends on the investor's risk profile (even the risk averse investors differ in terms of the magnitude of their risk aversion). Thus, without the knowledge of particular person's risk aversion level we can construct only so-called efficient set.

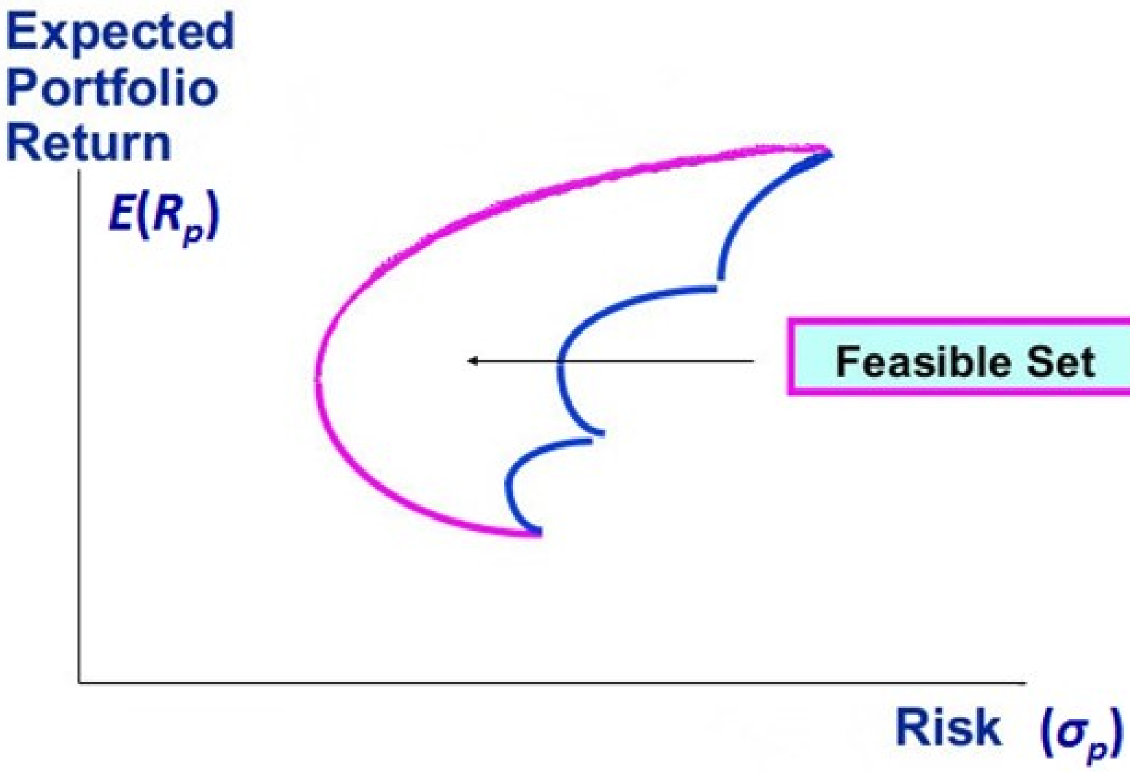

Chart 3.1: Feasible set of portfolios
Source: http://www.slideshare.net/Zorro29/chapter-7-slides-4682084

The efficient frontier is the line between the minimum variance portfolio and the maximum variance portfolio that traces out all attainable portfolios (asset combinations) that produce optimal/efficient portfolios. This set consists of portfolios which are Pareto efficient, Due to the assumption about investor; rationality, he or she will be interested only in those portfolios which fulfil following conditions at the same time:

- From the portfolios that have the same or higher return, the investor will prefer the portfolio with the lower risk,
- From the portfolios that have the same or lower risk level, the investor will prefer the portfolio with the highest rate of return.

Above mentioned conditions can be turn around so that particular portfolio is efficient if and only if there is no other portfolio with the lower risk delivering the higher or equal expected return and no other portfolio with the lower or equal risk delivering higher expected return. Selects efficient portfolios out of the feasible set is shown in Chart 3.2. In all previous analyses we assumed (in line with Markowitz model) two things: 1) short selling is not allowed; 2) only investments into risky assets are allowed.

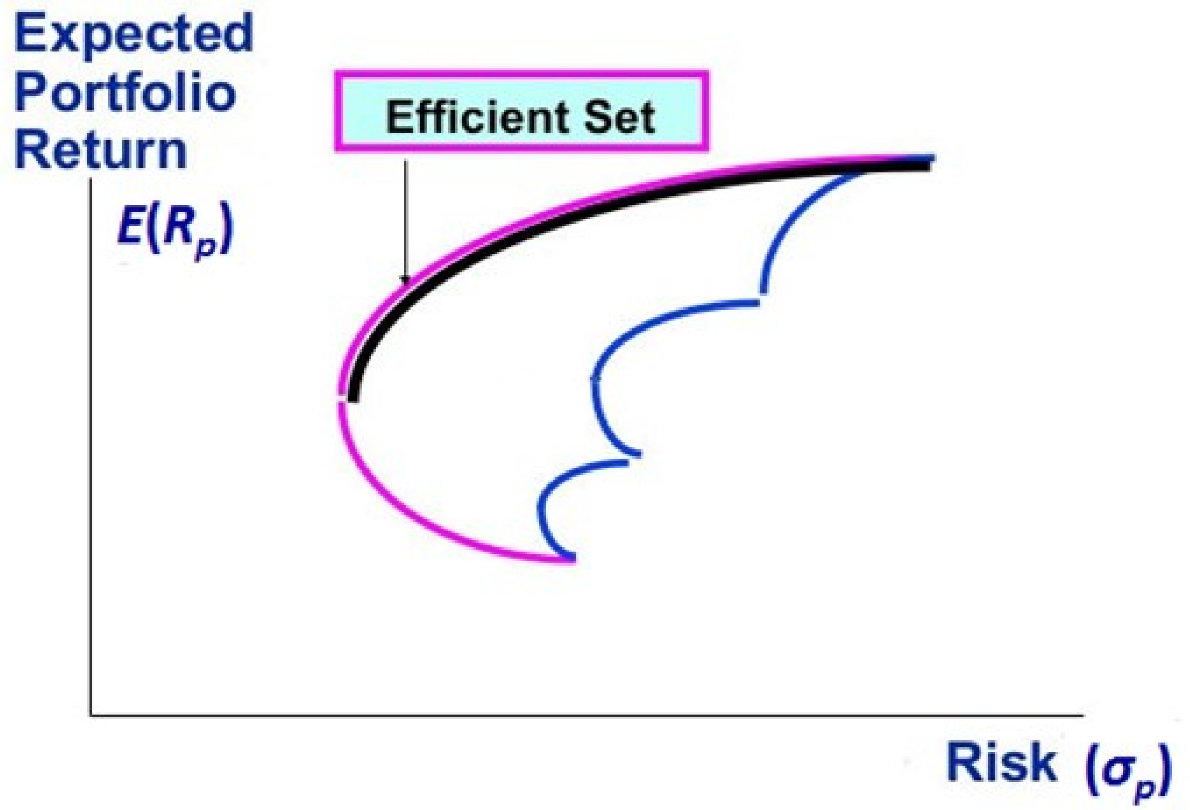

Chart 3.2: Efficient set of portfolios

Source: http://www.slideshare.net/Zorro29/chapter-7-slides-4682084

From the above, we find that in mean-variance analysis, expected return is plotted against risk (the standard deviation of asset returns) for a given portfolio. We generate random combinations of portfolio weights to produce a scatter plot of the expected return and risk for each portfolio (Chart 3.3). An efficient frontier is a set of optimal portfolios that offers the highest expected return for a specific level of risk, or the lowest risk for a given level of expected return. At least one portfolio can be created from all available investments for every point on the efficient frontier that has the expected risk and return corresponding to that point. It is not possible to have a portfolio lie above the efficient frontier. On the other hand, portfolios that lie below the efficient frontier are sub-optimal, because they do not offer sufficient return for the level of risk.

In Chart 3.3, the curve is known as the efficient frontier and contains the mean-variance efficient portfolios.

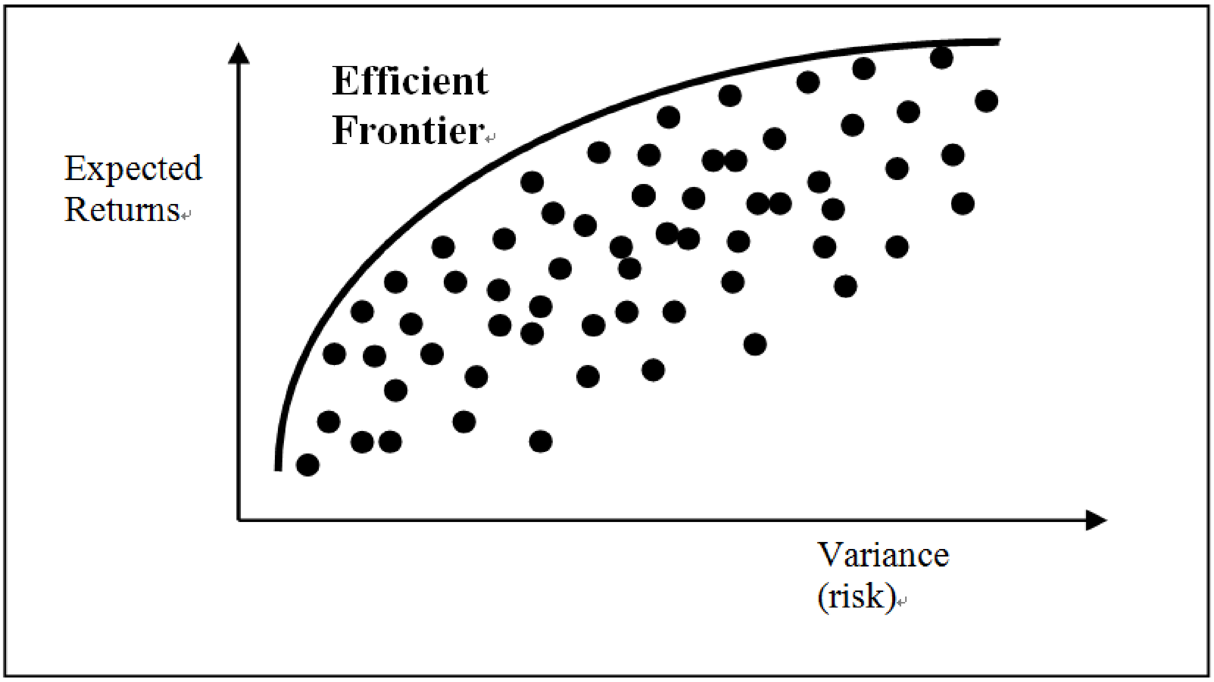

Chart 3.3: Efficient frontier of the mean-variance portfolio

The area below and to the right of the efficient frontier contains various risky assets (due to the discrete nature, the efficient frontier in the figure shown here is really a set of points, not a continuous line). The mean-variance efficient portfolios are combinations of these risky assets.

Modern portfolio theory is still largely based on the Markowitz model of mean-variance efficiency, or on assumptions related to it. An underlying assumption for this theory is that portfolio returns are normally distributed. While the mean-variance efficiency theory is still used throughout industry for securities portfolio selection, there is a growing body of evidence that suggests that actual portfolio returns are not normally distributed.

Rama Cont in his 2000 paper neatly summarizes a set of stylized facts (observed/assumed) common to time series properties of asset returns. In that paper, these stylized facts as a starting point to compare the time series properties of asset returns implied by popular stochastic volatility models are used. Cont lists the following a set of stylized statistical facts which are common to a wide set of financial assets.

1. Absence of autocorrelations - linear autocorrelations of asset returns are often insignificant except at very small time scales (high frequency data) for which microstructure effects come into play.

2. Heavy tails - the unconditional distribution of returns seems to display a power-law or Pareto-like tail, with a tail index which is finite, higher than two and less than five for most data sets studied. In particular this excludes stable laws with infinite variance and the normal distribution. However the precise form of the tails is difficult to determine.
3. Gain/loss asymmetry - one observes large drawdowns in stock prices and stock index values but not equally large upward movements.
4. Aggregational Gaussianity - as one increases the time scale t over which returns are calculated, their distribution looks more and more like a normal distribution. In particular, the shape of the distribution is not the same at different time scales.
5. Intermittency - returns display, at any time scale, a high degree of variability. This is quantified by the presence of irregular bursts in time series of a wide variety of volatility estimators.
6. Volatility clustering - different measures of volatility display a positive autocorrelation over several days, which quantify the fact that high-volatility events tend to cluster in time.
7. Conditional heavy tails - even after correcting returns for volatility clustering (e.g. via GARCH-type models), the residual time series still exhibit heavy tails. However, the tails are less heavy than in the unconditional distribution of returns.
8. Slow decay of autocorrelation in absolute returns - the autocorrelation function of absolute returns decays slowly as a function of the time lag, roughly as a power law with an exponent $\beta \in [0.2, 0.4]$. This is sometimes interpreted as a sign of long-range dependence.
9. Leverage effect - most measures of volatility of an asset are negatively correlated with the returns of that asset.
10. Volume/volatility correlation - trading volume is correlated with all measures of volatility.
11. Asymmetry in time scales - coarse-grained measures of volatility predict fine-scale volatility better than the other way round.[6]

[6] *CONT (2000)*

Overall, these are the effects which can influence the distribution of actual portfolio returns.

The minimum-variance portfolio is the portfolio (the combination of asset weights) that, given the particular return and risk characteristics of each asset, generates the lowest amount of risk achievable. In other words, the minimum variance portfolio specifies the asset weights that generate the lowest possible portfolio risk, without any additional constraints on the desired level of return or on the maximum or minimum extent to which an asset can enter into the portfolio.

The objective of this part to find the weights of assets in each portfolio, the manager of the stock portfolio has at the disposal stocks, there are American Express Company (AXP), The Boeing Company (BA), Caterpillar Inc. (CAT), Cisco Systems, Inc. (CSCO), Chevron Corporation (CVX), E. I. du Pont de Nemours and Company (DD), The Walt Disney Company (DIS), General Electric Company (GE), The Goldman Sachs Group, Inc. (GS) and The Home Depot, Inc. (HD) and so on. For given investment horizon expected (mean) returns $E(R_i)$ and the covariance matrix are from the result of the calculation of unconditional historical estimation of stock parameters.

The task of this part is:

(a) Select the optimal relative composition of different portfolios on the basis of the Markowitz model, including basic characteristics of expected returns and risks (described by standard deviation).

(b) The weights of assets have to be presented as well.

At first formulation the minimum risk portfolio, constrains show as below:

$$\sum_{i} x_i = 1. \tag{3.13}$$

The constraint shows as constraint (3.14) states that the sum of relative shares, and x_i is equal to one.

$$x_i \geq 0, for i = 1,2..., N. \tag{3.14}$$

The assumption allowed investing just the money amount we have held initially, the constraints exclude negativity, since short selling is not allowed there.

$$\sigma_p = \sqrt{\sum_i \sum_j x_i \cdot \sigma_{ij} \cdot x_j}. \tag{3.15}$$

By equation (3.15) defined the calculation of standard deviation for the optimal portfolio. Secondly formulation the maximum expected return portfolio, constrains show as below: Only then last constraint different from last formulation, it shows as:

$$E(R_i) = \sum_i x_i \cdot E(R_i), \tag{3.16}$$

3.3.2 Optimal efficient set

Respecting the fact, that there occur three distinct steps, we have to formulate three types of problems. The first step is to find the portfolio with the minimal risk; the second step is to find the portfolio with the maximal expected return. Subsequent steps consist in selecting the portfolios for interior points of the efficient set.

a) Formulation of the problem - the minimum risk portfolio

Objective function: $\sigma_p \to \min.$

Constraints: ○,1. $\sum_i x_i = 1,$

○,2 $x_i \geq 0, for\,i = 1,2...,N.$

The objective function expresses the minimal standard deviation of the portfolio we are looking for. The constraint 1 states that the sum of relative shares (percentages) x_i is equal to one. Hence, it is allowed to invest just the money amount we have held initially. The constraints 2 exclude negativity, since short selling is not allowed there. By equation 3.15, we define the calculation of the standard deviation for the efficient portfolio.

b) Formulation of the problem - the maximum expected return portfolio

Objective function: $E(R_p) \to \max.$

Constraints: ○,1. $\sum_i x_i = 1,$

○,2 $x_i \geq 0, for i = 1,2..., N.$

The objective function expresses the maximum expected return under given constraints. The constraints 1 and 2 are equivalent the previous problem. However, by equation 3.16, we define the calculation of an expected return for the optimal portfolio.

c) Formulation of the problem - the efficient set

Objective function: $\sigma_p \to \min.$

Constraints: ○,1. $\sum_i x_i = 1,$

○,2 $x_i \geq 0, for i = 1,2..., N,$

The aim of this problem is to select an efficient portfolio for a prespecified (generated) mean of the portfolio return. The objective function means the risk (standard deviation) minimization for efficient portfolios. The constraints 1 and 2 are defined in the same way as by previous problems. By the constraints 3 we ensure that the expected return of the particular efficient portfolio equals to the requested expected return.

3.4 Black's model

Black–Scholes–Merton model is a mathematical model of a financial market containing certain derivative investment instruments. The Black–Scholes model was first published by Fischer Black and Myron Scholes in their 1973 paper, "The Pricing of Options and Corporate Liabilities", published in the Journal of Political Economy. The model's assumptions have been relaxed and generalized in many directions, leading to a plethora of models that are currently used in derivative pricing and risk management. The Black–Scholes model assumes that the market consists of at least one risky asset, usually called the stock, and one riskless asset, usually called the money market, cash, or bond.[7]

The Black's model is used to determine optimal asset allocation in a portfolio, which takes the Markowitz Model one step further. The Black's model is the type of mean-variance model, by which we can invest only risky assets; however, the weight can be also negative. Hence, the difference between Markowitz and Black's model is the short sale of a stock is allowed in Black's model. We should also distinguish between limited (constrained) and unlimited short sales (on a basis of disposable resources).

3.4.1 Setting up the feasible and efficient set

The Black's model builds on the Markowitz model and it is hence important to understand Markowitz' model. Consider the Markowitz model, we know that the efficient frontier is the line in return/risk space that traces out all the portfolios for which we cannot obtain a higher level of return for a given level of risk, or alternatively for which we cannot obtain a lower level of risk for a given level of return. When we use Black's model analyze the composition of particular efficient stock portfolios with stated limit on short sales. We will see the possibility of short sales does not influence neither the composition nor the expected return and risk of the minimal risk portfolio.

The objective of this part is find the weights of assets in each portfolio, the manager of the stock portfolio has at the disposal stocks, American Express Company (AXP), The Boeing Company (BA), Caterpillar Inc. (CAT), Cisco Systems, Inc. (CSCO), Chevron Corporation (CVX), E. I. du Pont de Nemours and Company (DD), The Walt Disney Company (DIS), General Electric Company (GE), The Goldman Sachs Group, Inc. (GS) and The Home Depot, Inc. (HD) and so on. For given investment horizon expected (mean) returns $E(R_i)$ and the covariance matrix are from the result of the calculation of unconditional historical estimation of stock parameters.

[7] *BLACK et al. (1973)*

The task of this part is:

(a) Set up the optimal relative composition of Black's portfolios, assuming that the short selling is restricted by the initial financial resources and returns of portfolios.

(b) The weights of assets have to be presented as well.

At first formulation the minimum risk portfolio, constrains show as below:

$$\sum_i x_i = 1. \tag{3.17}$$

The constraint shows as constraint 3.17 states that the sum of relative shares, and x_i is equal to one.

$$x_i \geq -1, for i = 1,2..., N. \tag{3.18}$$

The assumption allowed investing just the money amount we have held initially, the constraints exclude smaller than minus 1, since shot selling is allowed there.

$$\sigma_p = \sqrt{\sum_i \sum_j x_i \cdot \sigma_{ij} \cdot x_j}. \tag{3.19}$$

By equation 3.19 defined the calculation of standard deviation for the optimal portfolio. Secondly formulation the maximum expected return portfolio, constrains show as below: Only then last constraint different from last formulation, it shows as:

$$E(R_i) = \sum_i x_i \cdot E(R_i), \tag{3.20}$$

After add the equation of 3.20 the constraints of maximum expected return is same to constraints (3.17) and (3.18). Finish calculate the all the efficient portfolios, you will find they are interval points.

3.4.2 Optimal efficient set

The procedure for finding the optimal set is similar to the Markowitz model. Thus, we first find the efficient portfolio with the minimal risk; subsequently we find the efficient portfolio with the maximal return. In this case, we add one more constraint in Black's model to avoid high volatility. The constraint is sum of absolute values of weights is smaller than 3.

a) Formulation of the problem - the minimum risk portfolio

Objective function: $\sigma_p \to \min.$

Constraints: ○,1. $\sum_i x_i = 1,$

○,2 $x_i \geq -1, for i = 1,2..., N.$

○,3 $\sum_i |x_i| \leq 3,$

The objective function expresses the minimal standard deviation of the portfolio we are looking for. The constraint 1 states that the sum of relative shares (percentages) x_i is equal to one. Hence, it is allowed to invest just the money amount we have held initially. The constraints 2 exclude the value smaller than -1, since short selling is allowed there. By equation (3.19), we define the calculation of the standard deviation for the efficient portfolio.

b) Formulation of the problem - the maximum expected return portfolio

Objective function: $E(R_p) \to \max.$

Constraints: ○,1. $\sum_i x_i = 1,$

○,2 $x_i \geq -1, for i = 1,2..., N.$

○,3 $\sum_i |x_i| \leq 3,$

The objective function expresses the maximum expected return under given constraints. The constraints 1 and 2 are equivalent the previous problem. However, by equation 3.20, we define the calculation of an expected return for the optimal portfolio.

c) Formulation of the problem - the efficient set

Objective function: $\sigma_p \to \min.$

Constraints: ○,1. $\sum_i x_i = 1,$

○,2 $x_i \geq -1, for i = 1,2..., N,$

○,3 $\sum_i |x_i| \leq 3,$

As before, we must formulate three types of problems. The formulation is almost identical to the preceding Markowitz model. The only difference is the change of constraint 2 into $x_i \geq -1$, for i=1, 2,…, N. This constrain characterize the assumption of short selling by the amount of disposable financial resources.[8]

3.5 Strategies based on portfolio optimization model

In this section, we can find the description of portfolio strategies under two financial models. Assume the investor have initial capital amount V_0 which equals to 1. In each period, they optimize the model investment and adjusted weights of assets in portfolios at the end of analyzed period. The objective of this part is calculating wealth of portfolio at the end of each period. The procedure of calculation in Markowitz and Black's model are introduced as below.

We can compute ex-post wealth (i.e. portfolio valuation) evolution and get the final wealth of the investment on maturity date.

The aim of this problem is to calculate the wealth of each month during the investment period by using selected optimal weights Since we have the weights of portfolios during the investment period under maximum expected return (w_i) and minimum variance strategy (w_i) which are described before, we can start calculate the real return of each month portfolio based on (3.21).

[8] *ZMEŠKAL et al. (2004)*

Final wealth calculation

$$R_{P,t} = \sum_{i=1}^{N} R_{i,t} \cdot w_{i,t} \tag{3.21}$$

$$W_{t+1} = W_t \cdot (1 + R_{P,t}). \tag{3.22}$$

where: $R_{i,t}$ is real return of the ith equity/stock, $w_{i,t}$ is the weight of the ith equity, $R_{P,t}$ is the real return of the portfolio at time t, W_t is initial wealth of investment and W_{t+1} is wealth in period t.

The second step is use initial investment amount 1 multiply real return of portfolios, so that real profit of each month are calculated, finally we can calculate the wealth from the first month which equals the real profit, the wealth after the second month should be accumulated.

As before, the formulation of Black's model is almost identical to the preceding Markowitz model. The only difference is the change of constraint 2 into $x_i \geq -1$, for i=1, 2,…, N. This constrain characterize the assumption of short selling is allowed which makes the weights of portfolios under maximum expected return and minimum variance are different. So follow the same process as Markowitz model, we also get two sets of portfolio wealth during investment period.

3.6 Performance measures

Having obtained the ex-post portfolio returns and ex-post wealth evolution we can analyze the performance of different strategies. However, there are many ways we can evaluate the strategies. There are some examples can be explained:

a) The mean annual return, which corresponds with the final value of the wealth, investors clearly want to maximize this measure;

b) The volatility of (daily) returns; the less volatile the returns were, the less risky the investment was;

c) The investors can also analyze the maximum drawdown (i.e. the maximum relative decline in the portfolio value over the analyzed period);

d) Some selected performance ratio of the reward and risk.

Based on the applied risk and reward measures there is a plenty of performance ratios. The examples are Sharpe ratio and others.

The maximum drawdown, Sharpe ratio will be explained as follows.

Maximum Drawdown

If we assume wealth path $W(t)$, we can measure the decline from the past maximal peak at time τ. this measure is called drawdown and can be computed as follows,

$$DD_{\tau} = 1 - \frac{W(\tau)}{\max\limits_{t \in (0,\tau)} W(t)}. \tag{3.23}$$

Note that (3.23) is stated in percentage – i.e. how big decline do we suffer at time τ related to the previous maximum wealth (the highest peak). However, we can extend the ratio so that we measure the maximum drawdown over the period (0, T).

$$MDD_{0,T} = \max_{\tau \in (0,T)} \left(1 - \frac{W(\tau)}{\max\limits_{t \in (0,\tau)} W(t)} \right). \tag{3.24}$$

The maximum drawdown (MDD) is the worst decline in the wealth over analyzed period, i.e. the maximum relative difference between the peak value and subsequent valley value. For further explanation see e.g. *Chekhlov et al. (2005)* or *Magdon-Ismail et al. (2004)* who studied the relationship between maximum drawdown and Geometric Brownian motion.

Sharpe Ratio

The portfolio that maximizes return relative to risk (the Sharpe Ratio) is the portfolio that lies on the tangency point between the Asset Allocation Line and the efficient frontier. The Sharpe ratio (also known as the Sharpe index, the Sharpe measure or the reward-to-variability ratio) is defined as the ratio between the excess expected return (i.e. the expected return minus risk-free rate, also known as risk premium) and its volatility. The ratio was developed by

William Forsyth Sharpe in 1998, see *Sharpe (1998)*, assuming following formula.

$$SR_{R,R_{RF}} = \frac{E(R - R_{RF})}{\sigma_{R-R_{RF}}} = \frac{E(R) - R_{RF}}{\sigma_R}. \tag{3.25}$$

The ratio was revised by *Sharpe (1994)* substituting risk-free rate by an applicable benchmark R_B, which changes in time.

$$SR_{R,R_B} = \frac{E(R - R_B)}{\sigma_{R-R_B}}. \qquad (3.26)^9$$

Final Wealth

Final wealth measures the value of all of the assets of worth owned by investors at the end of analyzed period of investment. It matters more than yield. Higher yield require higher levels of risk. All else equal, the longer the period in your assets holdings, the higher the yield. The final wealth is the value at the end of analyzed period due to the wealth of portfolios in each month is accumulated which are decreased in chapter 3.5. So we can get four value of final wealth of investment in Markowitz and Black's model under two types of strategies.

[9] *KRESTA (2015)*

4. Application of Portfolio Optimization Problem

Almost 60 years ago, Harry M. Markowitz published his modern portfolio Theory. Based on this theory, trillions of dollars were invested worldwide and its author with his followers received the 1990 Nobel Prize in Economics for it. The central idea of this theory lies in the fact that the market risk resulting from holding a financial instrument can be reduced by incorporating this instrument into the portfolio of instruments. The suitably composed portfolio may have a lower risk as a whole than what would correspond to the sum of risks of its individual components. The aim is then to find the best optimal portfolio.

In this chapter the solution of portfolio optimization problem is provided. The main objective of this chapter is analysis monthly portfolio value during investment period by application of Markowitz and Black's model which were introduced in previous chapter. In a further step, we analyze real return from different portfolios and calculate the final wealth for investor by using the weights of each asset we got. There are four parts of this chapter, historical data analysis, Markowitz model application, Black's model application and summary of comparison which describe optimal performance measures of the investment.

4.1 Historical data analysis

The objective of this part is disposal historical data which are downloaded from Yahoo Finance during period December 2004 to June 2014. We choose monthly adjusted closed price as the price of assets, and we can calculate the capital stock return (monthly) by equation (3.4) which introduced in Chapter 3.

4.1.1 Dow Jones Industrial Average dataset

The Dow Jones Industrial Average is a price-weighted average of 30 significant stocks traded on the New York Stock Exchange and the NASDAQ. The DJIA was invented by Charles Dow back in 1896.

The dataset we create consists solely of the stocks incorporated in one of the American stock market indices- Dow Jones Industrial Average (Henceforth DJIA). The dataset we want to create should cover the period from December 1, 2004 until June 2, 2014. Due to the lack of the historical data our data collections include mining 29 stocks from 30.

The practical implementation is depicted in Chart 4.1. The components on DJIA, downloading each of them from yahoo finance. Due to the fact that all the time series are of data are graphed according to the price of stocks without dealing with the missing data. Also it is possible download the illustration directly from yahoo finance.

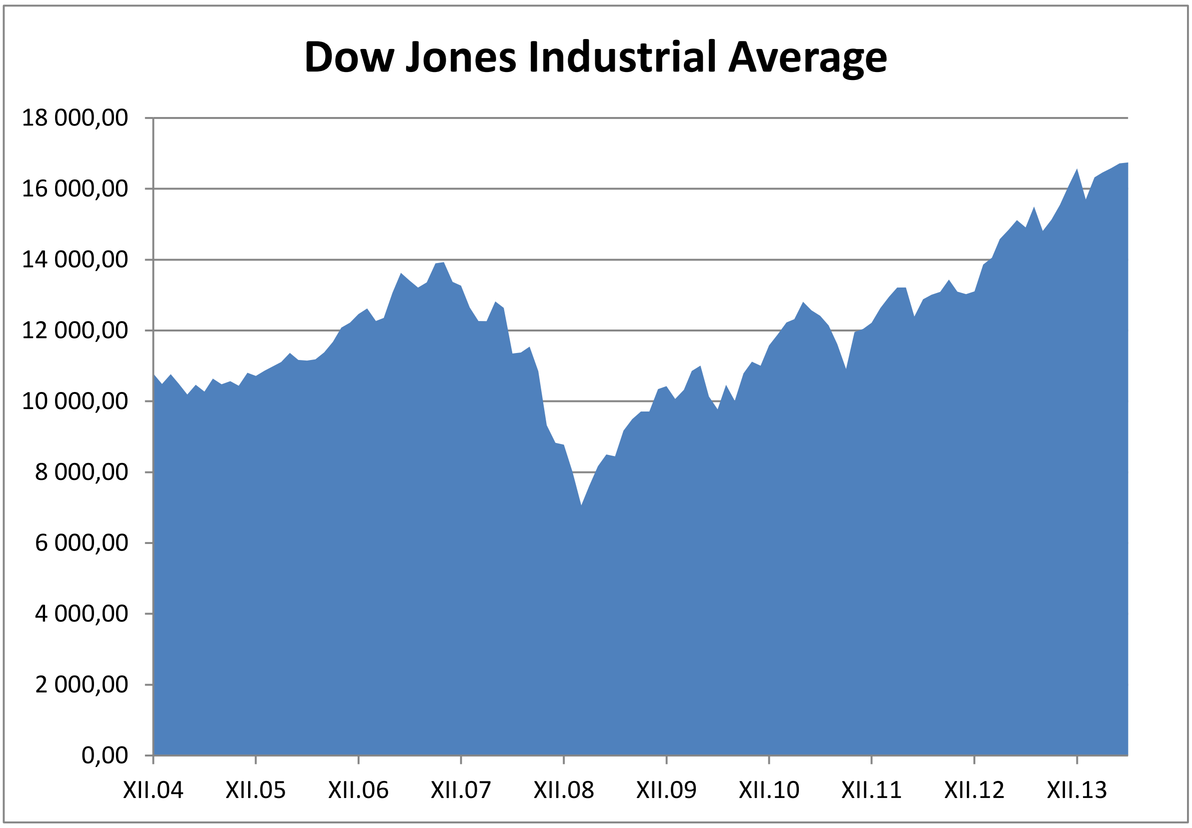

Chart 4.1: Dow Jones Industrial Average Index (DJIA)[10]

Source: http://finance.yahoo.com/echarts?s=%5EDJI+Interactive#

In Chart 4.1, we can find the index is appears rising trend with fluctuations during year 2004 to 2014, except 2008. As we know, the global financial crisis have strong influence to world economic, this year the index has a big drop which means the benchmark getting worse in that year due to financial crisis.

[10] *http://finance.yahoo.com/ DJI^*

4.1.2 Historical Stock Prices Collection

An example of Modern Portfolio Theory was run, using well-known financial assets over a period of ten years. The historical stock information of these financial assets has been presented in Table 4.1.

Table 4.1: The description (mean, variance, etc.) of returns

Stocks	Mean	Variance	Standard Deviation	Skewness	Kurtosis
AXP	0.0123	0.0127	0.1128	3.7490	30.1887
BA	0.0126	0.0056	0.0749	-0.6142	0.8788
CAT	0.0140	0.0098	0.0990	-0.0782	3.1738
CSCO	0.0060	0.0064	0.0798	0.0845	0.6074
CVX	0.0124	0.0034	0.0580	-0.3418	0.2340
DD	0.0085	0.0058	0.0762	0.1012	1.5501
DIS	0.0132	0.0041	0.0639	-0.2836	0.8912
GE	0.0036	0.0069	0.0831	-0.3207	1.8614
GS	0.0094	0.0086	0.0930	-0.1639	0.3242
HD	0.0099	0.0043	0.0654	-0.0852	-0.0004
IBM	0.0082	0.0030	0.0545	-0.6898	1.9198
INTC	0.0075	0.0053	0.0727	-0.2642	0.3592
JNJ	0.0077	0.0016	0.0398	-0.2994	0.9966
JPM	0.0094	0.0076	0.0872	-0.1854	0.7158
KO	0.0096	0.0020	0.0442	-0.3374	1.8412
MCD	0.0135	0.0019	0.0441	-0.2063	0.1649
MMM	0.0087	0.0033	0.0578	-0.3276	0.7967
MRK	0.0109	0.0044	0.0666	-0.2610	0.8787
MSFT	0.0080	0.0046	0.0681	0.1064	0.9160
NKE	0.0141	0.0043	0.0658	-0.1794	0.3768
PFE	0.0060	0.0033	0.0577	-0.2863	0.1686
PG	0.0063	0.0019	0.0438	-0.1152	0.3163
T	0.0085	0.0026	0.0506	-0.5981	0.3913
TRV	0.0116	0.0028	0.0525	0.1633	1.8457
UNH	0.0094	0.0065	0.0803	-0.9241	3.1271
UTX	0.0104	0.0030	0.0544	-0.2923	0.3043
VZ	0.0081	0.0027	0.0519	0.0039	-0.0776
WMT	0.0058	0.0021	0.0454	-0.1266	0.8592
XOM	0.0092	0.0028	0.0533	0.3929	1.9590

In this section, the adjusted closing values of the following financial assets were used: American Express Company (AXP), The Boeing Company (BA), Caterpillar Inc. (CAT), Cisco Systems, Inc. (CSCO), Chevron Corporation (CVX), E. I. du Pont de Nemours and Company (DD), The Walt Disney Company (DIS), General Electric Company (GE), The Goldman Sachs Group, Inc. (GS) and The Home Depot, Inc. (HD) and so on.

According to table 4.1, we can find the information about description of stocks returns collection. Here we list mean value, variance, standard deviation, skewness and kurtosis dataset of returns which are calculated based on the historical adjusted stocks prices. As we can see, the Nike, Inc. has highest average return of stock which is 0.0141 (1.41%), and the General Electric Company has the lowest (0.0036, 0.36%). The standard deviation describe that the stock's returns of Johnson & Johnson has a small difference between average values. It means the stock returns of the company are very stable. But American Express Company has a big value of standard deviation which means the stocks returns of the company's is volatile. Some companies have negative skewness, such as United Health Group Incorporated, International Business Machines Corporation and The Boeing Company and so on. Other companies have positive one, for instance: Verizon Communications Inc., Cisco Systems, Inc. and E. I. du Pont de Nemours and Company etc. It disclosed that the stock returns of companies like UNH are distribute in left of average returns and companies like VZ are distribute as opposite way. The results of kurtosis show that some companies have negative values which are platykurtic distribution. It means that their stock prices scattered and other companies have positive values which are leptokurtic distribution. This means these companies have concentrated stock prices with average value.

There are many differences between each company, some of them have good performance on stock price, but others are not. It is basic knowledge to do the further analysis.

4.2 Markowitz model analysis

Markowitz shows that investors under certain assumptions, theoretically, can build portfolios that maximize expected return given a specified level of risk, or minimize the risk given a level of expected return. The model is primarily a normative model. The objective for Markowitz has been not to explain how people select portfolios, but how they should select portfolios *(Sharpe, 1998)*. Even before 1952 diversification was a well-accepted strategy to lower the risk of a portfolio, without lowering the expected return, but until then, no thorough foundation existed to validate diversification. Markowitz' mean-variance portfolio model has remained to date the cornerstone of modern portfolio theory.

Efficient Frontier, also referred to as Markowitz Efficient Frontier, is a key concept of Modern Portfolio Theory (Efficient frontier/Money Terms). It represents the best combination of securities (those producing the maximum expected return for a given risk level) within an investment portfolio (Efficient Frontier). It describes the relationship between expected portfolio returns and the riskiness or volatility of the portfolio. It is usually depicted in graphic form as a curve on a graph comparing risk against the expected return of a portfolio. Portfolios lying on the 'Efficient Frontier' represent the best possible combination of expected return and investment risk.

In this part, we will use Markowitz model analysis the assets in our market, the aim of the analysis is to find the efficient set of the weights of portfolios. There are two different situations can be analyzed: a). Under the Markowitz model with maximum expected return; b). Under the Markowitz model with minimum variance. These two types of situation are introduced in previous chapter and the constraints of Markowitz are known.

4.2.1 Maximum expected return strategy

According to the previous chapter, we know that Markowitz model can help us to analyze the weights of all possible of portfolio. The procedure of analysis is described as follow.

Frist step is explained how to disposal historical data which are introduced in previous section. The second step is estimate the mean (expected) return of the assets by segment 12 months as first phase. Then we can use the same method to get all the result of mean return step by step. So we have all conditions to calculate the weights of every portfolio and the way of assets weights calculation is to apply the special module, *Tools* → *Solver*. In this model we should pay attention to the constraints of function in Markowitz which are described in previous chapter.

Finally we have 103 dataset of portfolios with weights of each asset. Actually, we can easier to get the result of weights, which is selecting the biggest value on return. This means the investor will pick the best performance of return in that period and invest all the money to one asset (highest value of mean return).

We can also get the optimal portfolio with the best expected return for each phase. There are dataset of portfolio value and we assume that the initial investment is 1 currency unit. There are 103 result of wealth of the investment from each portfolio in the period are come out by using equation (3.22) in chapter 3. The wealth value evolutions for all portfolios during period are shown in Chart 4.2.

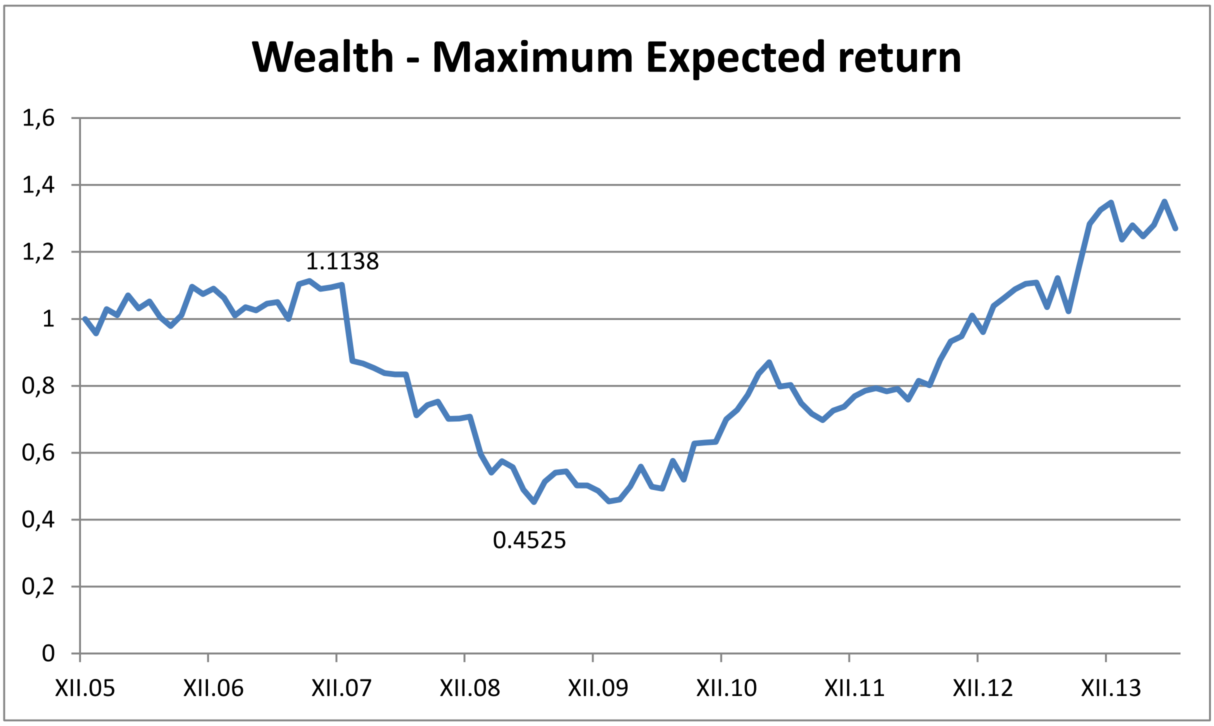

Chart 4.2: Wealth evolutions with maximum expected return

According to Chart 4.2, we can see that the trends of wealth is going up from year 2009 to 2014, it means the performance of chosen company is become better over this period. The investor can get profit from the end of year 2012 to 2014, because the wealth value s are bigger than initial investment amount. However, there is a big drop, which occurred during years 2007 to 2009, as we know, the maximum drawdown due to global financial crisis. The wealth value of portfolio drop from 1.1138 USD to 0.4524 USD and this maximum drawdown is -59.37%, so that the financial crisis causes great impact on the economic. On the other hand, investors only seek maximum return from investment. Though the investors have a big proportion to investing Walmart Store Company due to their good rate of return, it still cannot help anything.

4.2.2 Minimum variance strategy

In this part, we use Markowitz model with minimum variance to help us to analyze the weights of all possible portfolio. The procedure of analysis is described as follow.

It's similar to previous part and the first step is same. The second step is estimate the covariance of returns by segment 12 months as first phase. Then we can use the same method to get all the result of portfolio's variance step by step. So we have all conditions to calculate the weights of every portfolio and the way of assets weights calculation is to apply the special module, *Tools* → *Solver*. In this model we should pay attention to the constraints of function in Markowitz which are described in previous chapter under the objective function minimize the portfolio variance. Finally we have 103 dataset of portfolios with weights of each asset.

We can also get the optimal portfolio with minimum variance for each phase. There are dataset of portfolio value and we assume that the initial investment is 1 currency unit. There are 103 result of return of the investment from each portfolio in the period are come out by using equation (3.22) in chapter 3. The wealth value evolutions for all portfolios during period are shown in Chart 4.3.

From Chart 4.3, we can find that there is a rise trends from year 2005 to year 2007 and period 2009 to 2014. The maximum drawdown accrued in year 2008, which because of the financial crisis. The crisis effect the environment of economic very badly, we cannot help the investor increase the rate of return. The maximum wealth drop from 1.4400 USD to 0.9864 USD gives an information that investors better to try some measures to stop it, perhaps adjusted the portfolio structure to disperse risk. The economic situation recovered after crisis and the wealth growth kind of stable (big slope of the line). This is a very good news for investors that maybe they can expect better wealth in the future because this development trends.

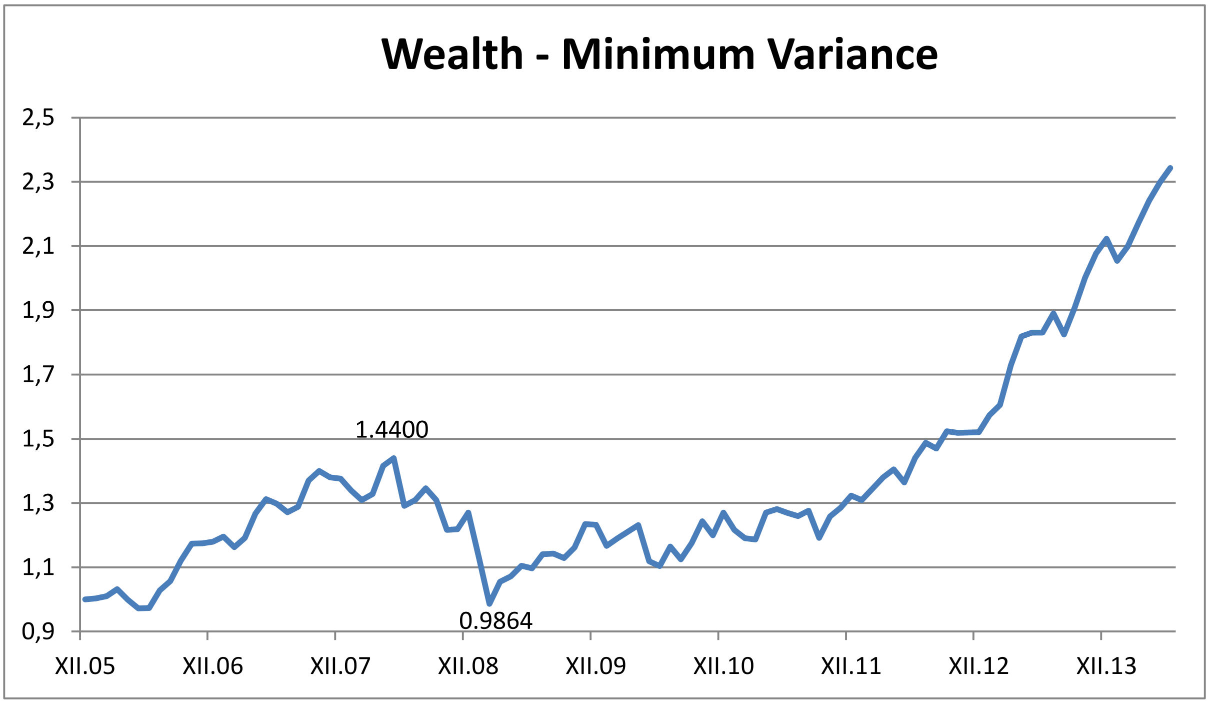

Chart 4.3: Wealth evolutions with minimum variance

4.3 Black's model analysis

The Black's model is the type of the mean-variance model, by which we can invest only into risky assets; however, the weight can be also negative. Hence, the short selling of a stock is allowed. We should also distinguish between limited (constrained) and unlimited short sales (on a basis of disposable resources).

In this part, we will use historical data of stocks to analyzing weights of each portfolio. It is very important information for investor to adjust the investment structure and predict the future return. We will use Black's model analysis the assets in our market, the aim of the analysis is to find the efficient set of the weights of portfolios. There are two different situations can be analyzed: a). Under the Black's model with maximum expected return strategy; b). Under the Black's model with minimum variance strategy. These two types of situation are introduced in previous chapter and the constraints of Black's are known.

4.3.1 Maximum expected return strategy

According to the Chapter 3, we know that Black's model can help us to analysis the weights of all possible portfolios. The procedure of analysis is described as follow which is similar as Markowitz model.

Frist step is explained how to disposal historical data which are introduced in previous section. The second step is estimate the mean (expected) return of the assets by segment 12 months as first phase. Then we can use the same method to get all the result of mean return step by step. So we have all conditions to calculate the weights of every portfolio and the way of assets weights calculation is to apply the special module, *Tools* → *Solver*. In this model we should pay attention to the constraints of function in Black's which are described in previous chapter.

Finally we have 103 dataset of portfolios with weights of each asset. Actually, we assume short selling is allowed in Black's model. For example, in the case, we have three copies of assets (the amount we have), two of them are investing the assets which are highest expected return and -1 share is investing the assets with lowest expected return which is short selling. The constraint that the sum of absolute values of weights is smaller than three means total short position is always lower than the amount we have. The investor will pick the best performance of return in that period and invest all money to one asset (highest value of mean return).

We can also get the optimal portfolio with best expected return for each phase. There are dataset of portfolio value and we assume that the initial investment is 1 currency unit. There are 103 result of wealth of the investment from each portfolio in the period are come out by using equation (3.22) in chapter 3. The wealth value evolutions for all portfolios during period are shown in Chart 4.4.

According to the Chart 4.4, we can find that the trends of the wealth of stock portfolios in Black's model is fluctuated and decreased during period 2005 to 2009. This is an awful sign for the investor, the wealth is less than 1 USD (initial investment capital) means the investor loss their money during investment period, even did not recovery till the end of period. The main reason is global financial crisis give a shock to economic. There is an increase trends occurred from 2010 to 2014, because of the corporates which are invested with big proportion have good economic benefits and try to recovery after big recession. For instance: The Boeing Company, Caterpillar Inc. company and The Home Depot, Inc. company.

Chart 4.4: Wealth evolutions with maximum expected return

Speaking of BA Company, there is an event happened in 2013 which is Biological fuel plan. On January 31, 2013, Boeing has received a $13.6 million contract, to upgrade the U.S. air force “Combat Survivor Evader Lacator, (CSEL)”, as well as to support the CSEL ultra-high frequency (UHF) base station. The company developed pretty well since 2013 and brings more benefit for investors. Maybe these are the reasons of increases during that period. It brings good sign for investor, but it seems does not worked as the trends shown in the end.

4.3.2 Minimum variance strategy

In this part, we use Black’s model with minimum variance to help us to analysis the weights of all possible of portfolio. The procedure of analysis is described as follow.

It’s similar to previous part and the first step is same. The second step is estimate the variance of returns by segment 12 months as first phase. Then we can use the same method to get all the result of portfolio's variance step by step. So we have all conditions to calculate the weights of every portfolio and the way of assets weights calculation is to apply the special module, *Tools* → *Solver*. In this model we should pay attentions to the constraints of function in Black's which are described in previous chapter under the objective function minimize the portfolio variance. Finally we have 103 dataset of portfolios with weights of each asset.

We can also get the optimal portfolio with minimum variance for each phase. There are dataset of portfolio value and we assume that the initial investment is 1 currency unit. There are 103 result of wealth of the investment from each portfolio in the period are come out by using equation (3.22) in chapter 3. The wealth value evolutions for all portfolios during period are shown in Chart 4.5.

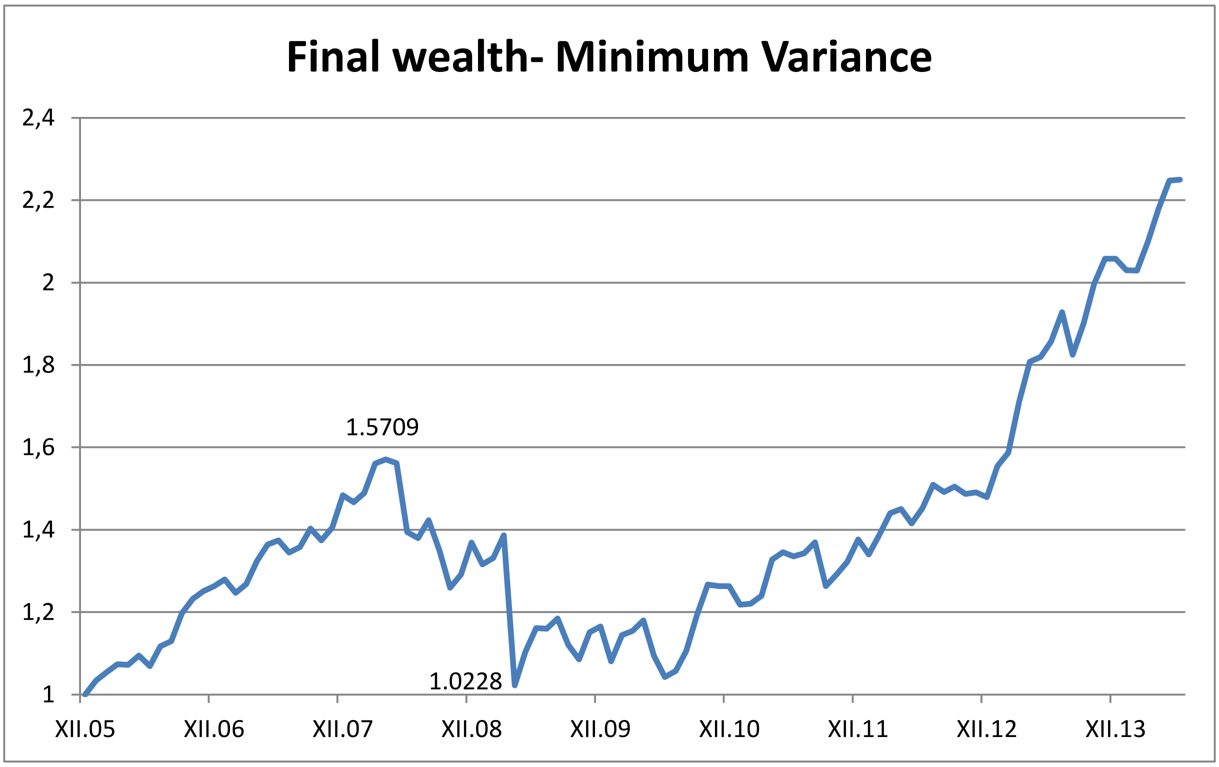

Chart 4.5: Wealth evolutions with minimum variance

From Chart 4.5, we can find that the trend under minimum variance strategy is similar to Markowitz model. The wealth increases from year 2005 to year 2007 and 2009 to 2014. Due to the financial crisis, the biggest drop accrued in year 2008 from 1.5709 USD to 1.0228 USD, which the maximum drawdown is -34.89%. The the crisis effect the environment of economic very badly, however, the wealth of portfolios are all over than 1 (initial investment capital) is good situation. The main point is adjusting portfolio structure for good and get though recession period.

4.4 Portfolio manager recommendation

In this section, the summaries of financial model analysis and performance measures results are interpreted. There are some recommendations for investor, such as which model is better for analyzing portfolios, which strategy can bring more wealth for investors and which performance index is good for measurement.

4.4.1 Comparison of models

The comparison of Markowitz model and Black's model can be found in this section. We can disclose the difference between these two models under same strategy, both in opposite way.

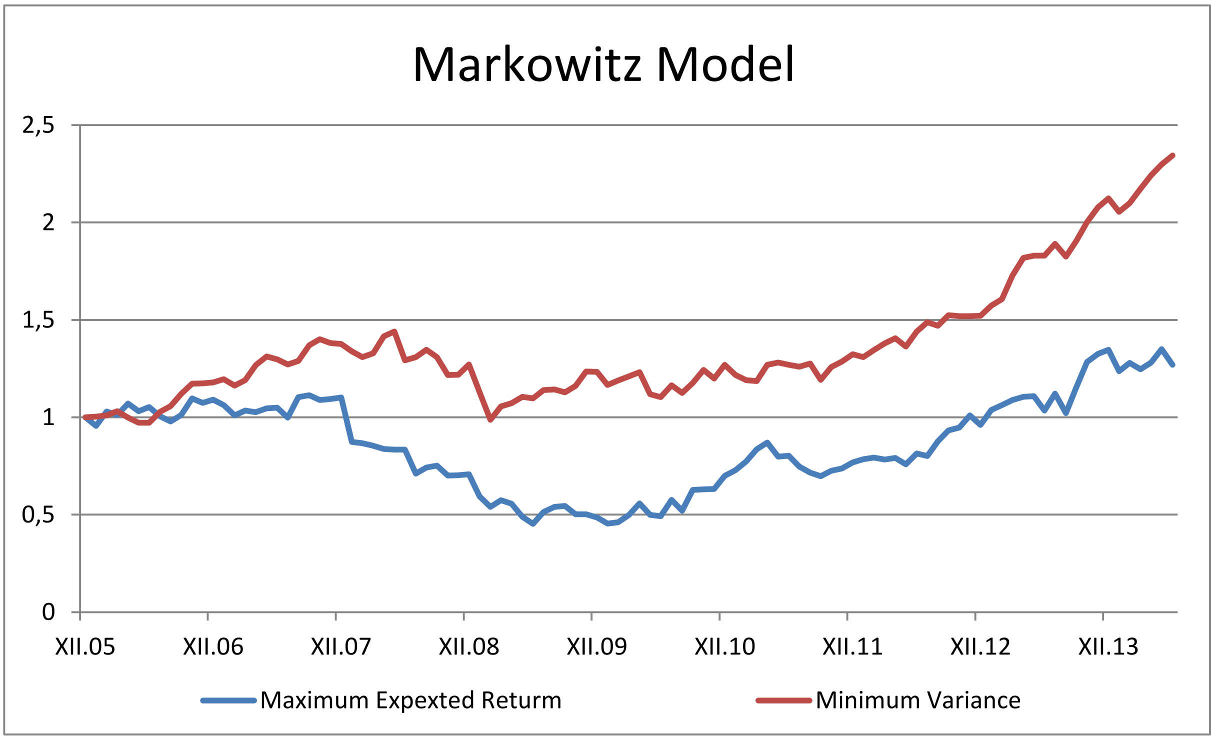

Chart 4.6: Comparison between Maximum expected return and Minimum variance strategy

First of all, the comparison between maximum expected return and minimum variance strategy in Markowitz model are illustrated as Chart 4.6. We can find that these two strategies have similar trends, but as we can see, the minimum variance strategy can bring more wealth for investors. Compared with two strategies, we have -31.50% maximum drawdown under minimum variance which is lower than -59.37% (under maximum expected return strategy). In this case, it is better for investors to choose the strategy minimize variance of portfolio, because under this strategy will reduce the risk of investment efficiently.

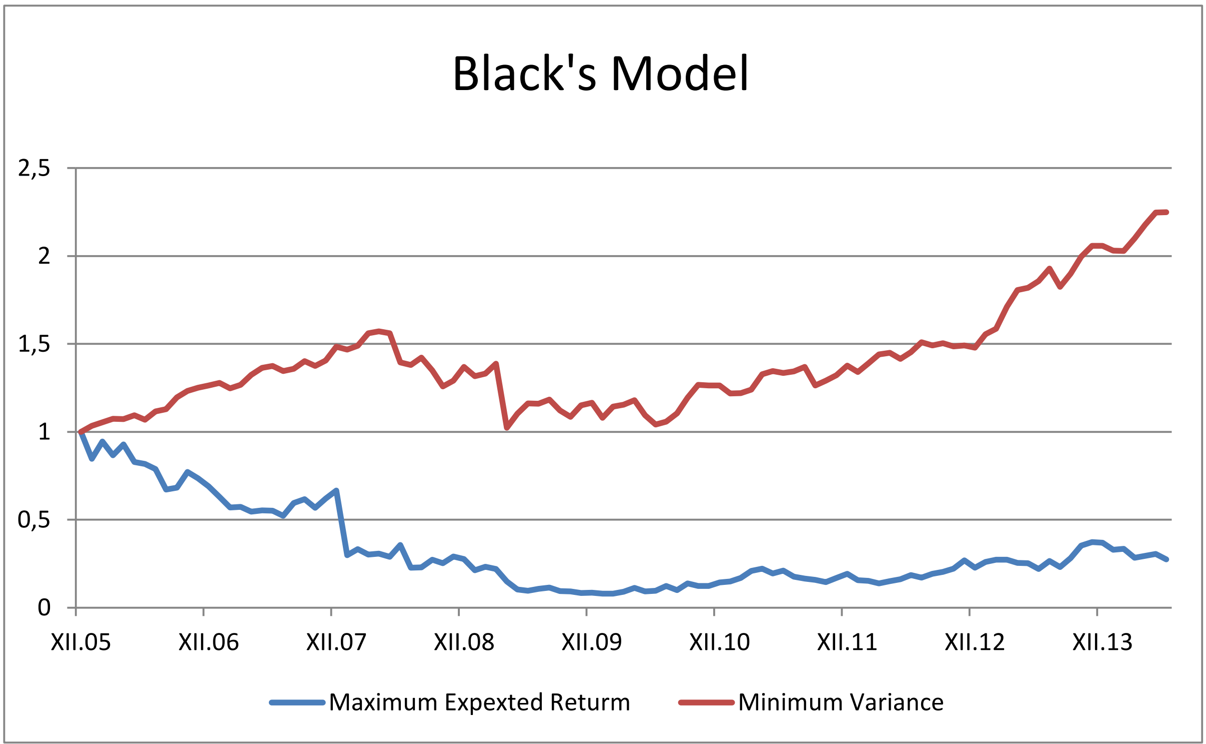

Chart 4.7: Comparison between Maximum expected return and Minimum variance strategy

The Chart 4.7 represents these two strategies in Black's model, which are illustrated as we can see. The trends of two lines are kind of different; there is a bad situation under maximum expected return strategy based on Black's assumption that short selling is allowed. Compared final wealth of investment between these two strategies, we can see that the red line brings more wealth than the blue one and the red line always shows higher than 1 (initial investment amount), rather than the blue one. On the other hand, the developments at the end of period are totally opposite; it seems that the red line maybe increases in the future and the blue one decreases.

Generally speaking, minimize variance strategy of portfolios both in Markowitz and Black's model is better choice.

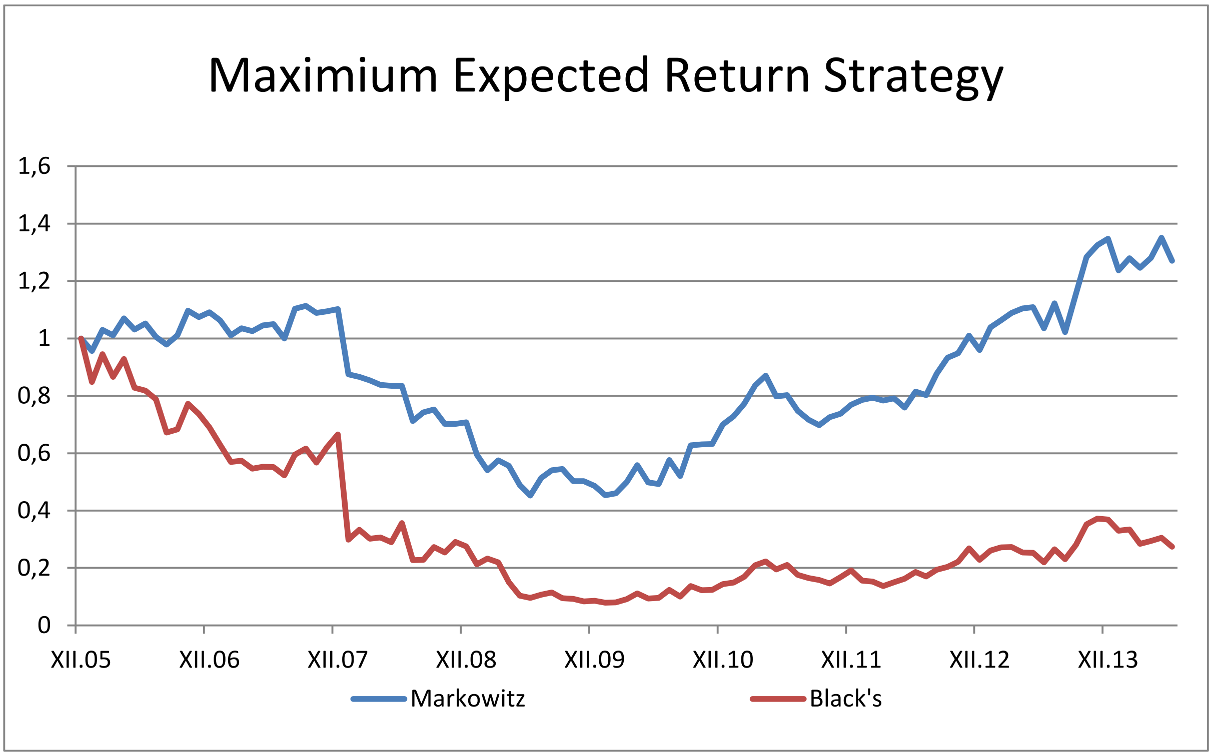

Chart 4.8: Comparison between Markowitz and Black's model

After the analysis, we can get the wealth trends of two models under maximum expected return strategy which graphic in Chart 4.8. The results show that the trends of both models approximately same which are decreased over crisis period and recovered after that. Besides, Markowitz has higher wealth than Black's during period and also have bigger slope of increase which means the speed of recovery is more quickly. So, we get the result that Markowitz .have better situation under maximum expected return strategy than Black's, which should be chosen.

In Chart 4.9, the comparison between two models is illustrated under minimum variance. As we can see, the trends lines of these two models are very similar, almost same. Which one is better is not apparent, so it is better to choose Markowitz. Because we can find the trends of Markowitz at the end of period has a small advantage than Black's. According to the principle, we need consider the future trends of investment; it seems that we can predict the Markowitz has more wealth in the future than Black's. So that Markowitz model should be chosen.

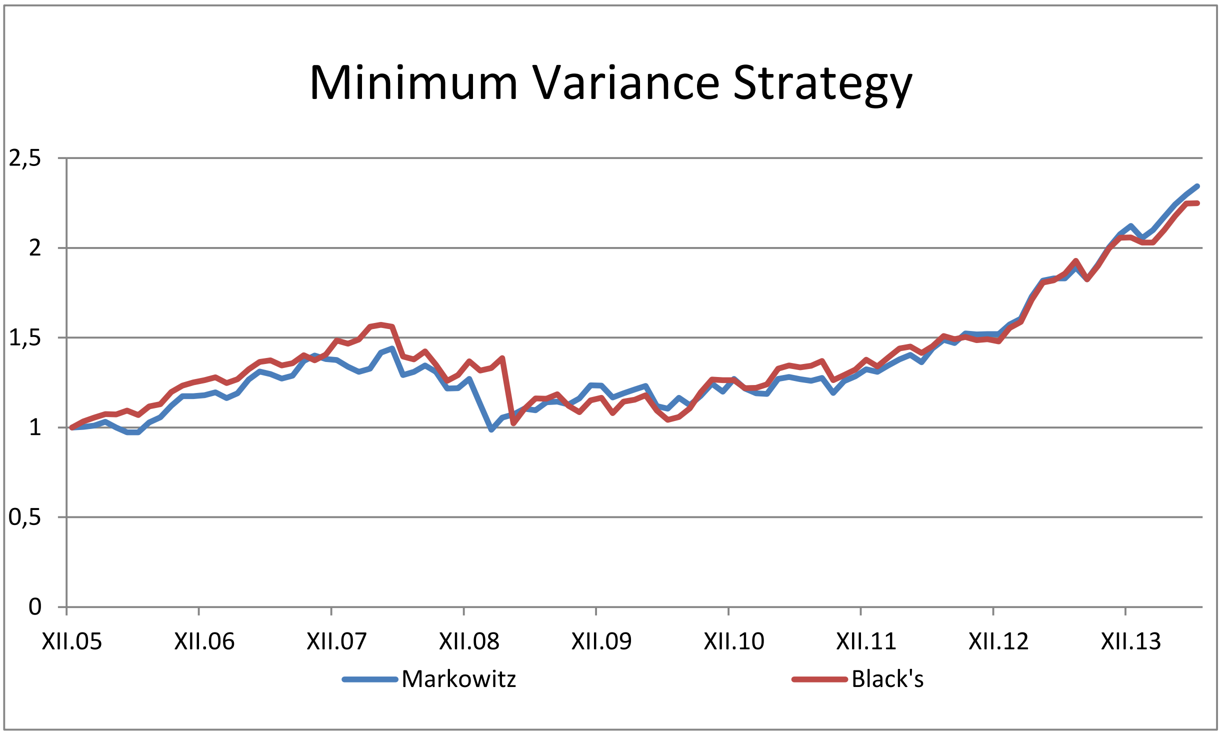

Chart 4.9: Comparison between Markowitz and Black's model

Overall, the comparison results are shown that the minimum variance strategy and Markowitz model are chosen in different situations. So we can consider that Markowitz model under minimum variance strategy are best choice for investors as useful recommendations.

4.4.2 Best decision-making of investment

From this section, the analysis of performance between different models and different strategies are provided. There are three types of measures: maximum drawdown, Sharpe ratio and final wealth. It is illustrated in Table 4.2 as below:

Table 4.2 exposed the performance measures. We can find that the result of final wealth is same as previous analysis, which investor can get highest wealth (2.3430 USD) under minimum strategy by using Markowitz model. Compared Maximum drawdown, we can get same result that Markowitz model under minimize variance strategy is the best (-31.50%) and Black's model under maximum expected return is the worst (-92.09%). In this way, we have lowest value of maximum drawdown which reduced the risk of investment and explained the wealth of the portfolios is stable. The Sharpe ratio measures excessive return of unit risk, which means higher Sharpe ratio, is better. As we can see, Markowitz model under minimize variance strategy also is the best (0.2310).

Table 4.2: Comparison of financial models

Model	Markowitz Model		Black's Model	
Measure	**Max-expected return**	**Min-variance**	**Max-expected return**	**Min-variance**
Final wealth	1.2702	2.3430	0.2744	2.2495
Maximum drawdown	-59.37%	-31.50%	-92.09%	-34.89%
Sharpe ratio	0.0681	0.2310	-0.0079	0.2001

After analysis of performance measures, the investor can find which model and strategies is suitable for them, and make best decision of investment. Investors with maximal risk-aversion should, under given assumptions, select the efficient stock portfolio, while investors whose version to risk is minimal, should choose the portfolio which has the minimal share of the riskless asset.

Generally speaking, according to analysis we have the result of how to recommended investment to choose the best way to analysis and manage their assets. All these analysis of models and strategies even the performance measures, we got one conclusion that using Markowitz model under minimum variance strategy is the best choice for investor to analysis the portfolio, which can reduce the risk of portfolio and maximize return compared with other. In a word, we recommended the investors to optimal portfolio by Markowitz model under minimum variance strategy.

5. Conclusion

Ongoing integration and globalization of financial market, investment become a role of importance and indispensable for people's life. In the global market, there are more choices for investors, so it is necessary to know how to set up assets portfolio. About investment, there is famous saying "don't put all eggs in one basket", which point the importance of diversification during procedure of investment. The objective of diversification is to maximize returns and minimize risk by investing in different assets that would each react differently to the same event. For instance, negative news related to the European debt crisis generally causes the stock market to move significantly lower. At the same time, the same news has had a general positive impact on the price of certain commodities such as gold. Accordingly, any savvy investor who is risk-averse will diversify to some degree. The best way to achieve this goal is optimal portfolio of assets by using financial model under variables strategies.

The thesis is focused on assess the optimal portfolio, based on result of calculation by using financial modeling, specifically stock portfolios efficient set and its optimization. As the financial modeling is to provide the information about investor's decision-making of their investment, it is very important. The goal of the thesis is to perform ex-post analysis of portfolio different optimization problems, which arise from different risk attitudes.

Markowitz model and Black's model under both maximum expected return and minimum variance strategy are described. In short, we found out that: Markowitz model is similar with the Black's model; the investors are choosing the portfolio which has the highest expected return which is based on the risk attitude of the investors we assumed. In my opinion, decision making depends on many parameters, not only on the risk level and expected return but other relevant factors about stocks. There are more model can be used to assess the portfolios, such as combination of efficient sets and value at risk methodology for a portfolio of stocks. On the other hand, the investors have different risk attitude, which can be indispensable impact factor. Anyway, there are optimal portfolio decision-making recommendations for investors after more analysis.

As the above analysis shows, the expected returns of portfolios are seldom normally distributed. This creates the need for optimization methods that do not rely solely on theory derived from Markowitz's model. Investors need to assess multiple scenarios in order to select a portfolio that aligns with their strategy and risk profile. By using a methodology and a tool that clearly communicates the performance of the portfolio in each scenario, the investors can make better decisions. Our results show that, through the use of more performance measures, we can guide our search towards improvements in the performance of the desired portfolio of projects. Such as: analyzing data by Markowitz model under minimize variance strategy can bring highest wealth for investors, which is 2.3430 USD (134.3%); maximum drawdown gives the information that Markowitz under minimum variance have better situation than other models. The value of maximum drawdown is -31.50% (lowest) means the best; besides, the highest value of Sharpe ratios (0.2310) should be chosen which also use Markowitz model under minimum variance strategy.

We analyze the price time series for a ten years period from December 1, 2004 to June 2, 2014. This period includes the 2007-2008 US subprime crises that affected the financial markets of all over the world. We can say the period we have selected for our analysis is characterized by a high volatility and a negative trend caused by adverse economic events. The results also show that the wealth has badly impact, which represented in both model and strategy. However, compared with all situations we still can find better performance of investment. So we figure out that adjusted weights of portfolio are useful and investors should get through the recession, everything will become better. Consider the result of these two financial models, the first recommendation to investor is choosing minimum variance of portfolios by using Markowitz model.

In the thesis, we can find some problems during analysis process. For example, the main drawback with modern portfolio theory is that it relies on historical data. This is because, when working with stocks, their past trends have no influence on future trends. However, it is inconvenient to analysis with erroneous data or missing data. Anyway, to make some conclusion we can say that investors have different financial models and strategy to analysis portfolio and make suitable investment decision.

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List of Abbreviations

AXP: American Express Company

BA: The Boeing Company

CAT: Caterpillar Inc.

CSCO: Cisco Systems, Inc.

CVX: Chevron Corporation

DD: E. I. du Pont de Nemours and Company

DIS: The Walt Disney Company

GE: General Electric Company

GS: The Goldman Sachs Group, Inc.

HD: The Home Depot, Inc.

IBM: International Business Machines Corporation

INTC: Intel Corporation

JNJ: Johnson & Johnson

JPM: JPMorgan Chase & Co.

KO: The Coca-Cola Company

MCD: McDonald's Corp.

MMM: 3M Company

MRK: Merck & Co. Inc.

MSFT: Microsoft Corporation

NKE: Nike, Inc.

PFE: Pfizer Inc.

PG: The Procter & Gamble Company

T: AT&T, Inc.

TRV: The Travelers Companies, Inc.

UNH: UnitedHealth Group Incorporated

VZ: Verizon Communications Inc.

WMT: Wal-Mart Stores Inc.

XOM: Mobil Corporation

MPT: Modern Portfolio Theory

c.u.: currency unit

CE: certainty equivalent

$E(U(W))$: expected value of the utility (expected utility) of the uncertain payment

$E(W)$: expected value of the uncertain payment

$U(CE)$: utility of the certainty equivalent

$U(E(W))$: utility of the expected value of the uncertain payment,

$U(W_0)$: utility of the minimal payment

$U(W_1)$: utility of the maximal payment

W_0: minimal payment

W_1: maximal payment and *RP* is risk premium.

TVM: time value of money

$E(R_p)$: the portfolio expected return

σ_p^2 : portfolio variance

σ_p : standard deviation

Q: covariance matrix of returns

$R_{i,t}$: the return from month $t-1$ to month t of asset i

$P_{i,t}$: the price at month t of asset i

$P_{i,t-1}$: the price at month t-1 of asset i.

ρ_{ij} : the correlation between asset i and j

w_i : the fraction of the portfolio invested in asset i

U: the expected utility of returns

$E(R_i)$: the expected return of asset i

σ_{ij} : the covariance between asset i and j

σ_i : the standard deviation of asset i

σ_j : the standard deviation of asset j

$R_{P,t}$: real retrun rate in period t

V_0: initial investment amount

W_t : initial wealth of investment

W_{t+1} : wealth in period t.

CSEL: Combat Survivor Evader Lacator,

UHF: ultra-high frequency

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MENG TING REN

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List of Annexes

Annex 1

Historical stock price (example)

Date	AXP	BA	CAT	CSCO	CVX	…	UNH	UTX	VZ	WMT	XOM
2004/12/1	42.26	41.18	38.08	17.51	37.50	…	40.81	41.44	21.97	42.91	40.5
2005/1/3	40.08	40.25	34.95	16.35	38.85	…	41.21	40.37	19.49	42.57	40.77
2005/2/1	40.69	43.93	37.28	15.79	44.64	…	42.26	40.22	19.7	41.92	50.26
2005/3/1	38.69	46.72	35.87	16.22	41.93	…	44.23	40.94	19.44	40.82	47.32
2005/4/1	39.69	47.57	34.70	15.65	37.39	…	43.83	40.96	19.83	38.41	45.28
2005/5/2	40.56	51.28	37.09	17.58	39.02	…	45.06	43.15	19.6	38.6	44.85
2005/6/1	40.18	52.96	37.56	17.29	40.57	…	48.36	41.53	19.14	39.39	45.86
2005/7/1	41.51	52.97	42.69	17.36	42.09	…	48.51	41.01	19.18	40.33	46.88
2005/8/1	41.69	53.98	43.94	15.97	44.88	…	47.76	40.62	18.33	36.86	48.03
2005/9/1	43.36	54.73	46.52	16.24	47.31	…	52.12	42.11	18.32	35.93	50.95
2005/10/3	43.02	52.06	41.83	15.82	41.71	…	53.69	41.66	17.88	38.79	45.02
2005/11/1	44.45	55.13	45.96	15.90	42.23	…	55.52	43.92	18.15	39.81	46.77
2005/12/1	44.48	56.79	45.96	15.52	41.83	…	57.63	45.6	17.09	38.49	45.27
2006/1/3	45.44	55.23	54.23	16.83	43.75	…	55.11	47.61	18.2	37.92	50.57
2006/2/1	46.68	59.02	58.37	18.35	41.95	…	54.01	47.9	19.37	37.3	48.1
2006/3/1	45.53	63.27	57.35	19.64	43.05	…	51.84	47.46	19.58	38.99	49.31
2006/4/3	46.73	67.76	60.68	18.99	45.32	…	46.16	51.43	19.21	37.17	51.11
2006/5/1	47.20	67.82	58.45	17.84	44.79	…	40.79	51.4	18.15	40.13	49.6
…	…	…	…	…	…	…	…	…	…	…	…
2013/2/1	60.75	73.7	87.45	19.64	109.29	…	51.76	86.75	42.64	67.39	84.67
2013/3/1	65.94	82.27	82.33	19.67	110.85	…	55.62	89.51	45.04	71.71	85.20
2013/4/1	67.07	87.6	80.67	19.85	113.83	…	58.26	87.46	49.92	74.48	84.14
2013/5/1	74.22	95.38	81.75	22.89	115.44	…	60.88	91.43	44.89	72.15	86.13
2013/6/3	73.29	98.68	78.59	23.1	111.29	…	63.94	89.54	46.62	71.81	86.02
2013/7/1	72.54	101.24	79.55	24.45	118.39	…	71.13	101.71	46.28	75.14	89.26
2013/8/1	70.71	100.56	79.19	22.27	114.19	…	70.05	96.93	44.32	70.78	83.56
2013/9/3	74.26	113.7	80.02	22.39	115.21	…	70.18	104.40	43.66	71.74	82.48
2013/10/1	80.68	126.28	80.54	21.72	113.75	…	66.90	102.88	47.78	74.44	85.91
2013/11/1	84.63	130.38	81.74	20.45	117.07	…	73.00	107.94	46.94	78.57	90.22
2013/12/2	89.49	132.56	87.74	21.59	119.44	…	74.08	110.80	46.49	76.77	97.67
2014/1/2	84.07	121.65	91.32	21.25	106.74	…	71.11	111.02	45.92	72.86	88.95
2014/2/3	90.26	125.91	94.3	21.14	111.26	…	76.02	114.54	45.50	72.87	93.57
2014/3/3	89.03	122.56	96.63	21.75	114.71	…	80.96	114.36	45.49	75.04	94.94
2014/4/1	86.68	126.01	103.1	22.61	121.09	…	74.09	115.82	45.19	78.26	99.54
2014/5/1	90.71	132.83	100	24.08	119.46	…	78.63	114.32	48.31	75.84	98.38
2014/6/2	94.05	124.96	106.3	24.31	127.01	…	81.10	113.56	47.31	74.16	98.53

Annex 2

Expected return of investment period (example)

Date	AXP	BA	CAT	CSCO	…	UTX	VZ	WMT	XOM
2005/12/1	0.005	0.028	0.018	-0.009	…	0.008	-0.020	-0.008	0.012
2006/1/3	0.011	0.027	0.040	0.004	…	0.014	-0.005	-0.009	0.022
2006/2/1	0.012	0.026	0.041	0.014	…	0.015	-0.001	-0.009	-0.002
2006/3/1	0.014	0.026	0.043	0.018	…	0.013	0.001	-0.003	0.005
2006/4/1	0.014	0.031	0.050	0.018	…	0.020	-0.002	-0.002	0.012
2006/5/2	0.013	0.024	0.041	0.003	…	0.015	-0.006	0.004	0.010
2006/6/1	0.012	0.020	0.042	0.003	…	0.020	0.002	0.002	0.009
2006/7/1	0.008	0.016	0.027	-0.004	…	0.019	0.004	-0.006	0.016
2006/8/1	0.008	0.012	0.019	0.022	…	0.021	0.011	0.002	0.014
2006/9/1	0.010	0.015	0.014	0.024	…	0.019	0.016	0.012	0.008
2006/10/3	0.014	0.020	0.016	0.030	…	0.023	0.018	0.006	0.023
2006/11/1	0.012	0.024	0.009	0.040	…	0.017	0.016	-0.002	0.027
2006/12/1	0.015	0.022	0.008	0.043	…	0.011	0.026	0.001	0.029
2007/1/3	0.010	0.025	-0.002	0.034	…	0.015	0.024	0.005	0.017
2007/2/1	0.006	0.018	-0.008	0.024	…	0.012	0.016	0.008	0.018
2007/3/1	0.007	0.013	-0.003	0.017	…	0.012	0.017	0.002	0.021
2007/4/3	0.012	0.011	-0.001	0.024	…	0.007	0.020	0.008	0.022
2007/5/1	0.017	0.018	0.009	0.029	…	0.012	0.036	0.001	0.029
…	…	…	…	…	…	…	…	…	…
2013/1/2	0.015	0.002	-0.005	0.010	…	0.013	0.017	0.014	0.009
2013/2/1	0.016	0.005	-0.014	0.011	…	0.010	0.021	0.018	0.006
2013/3/1	0.015	0.015	-0.013	0.006	…	0.013	0.026	0.020	0.006
2013/4/1	0.013	0.018	-0.012	0.010	…	0.012	0.029	0.026	0.006
2013/5/1	0.028	0.033	0.001	0.038	…	0.023	0.018	0.013	0.014
2013/6/3	0.023	0.030	0.001	0.035	…	0.020	0.016	0.008	0.007
2013/7/1	0.023	0.033	0.002	0.045	…	0.033	0.013	0.006	0.009
2013/8/1	0.020	0.035	0.000	0.022	…	0.022	0.013	0.003	0.003
2013/9/3	0.026	0.047	0.000	0.022	…	0.030	0.007	0.003	-0.002
2013/10/1	0.034	0.056	0.002	0.027	…	0.029	0.016	0.005	0.001
2013/11/1	0.038	0.053	0.002	0.013	…	0.030	0.015	0.013	0.008
2013/12/2	0.041	0.054	0.004	0.015	…	0.031	0.016	0.015	0.016
2014/1/2	0.033	0.048	-0.001	0.009	…	0.025	0.013	0.008	0.005
2014/2/3	0.035	0.047	0.007	0.008	…	0.025	0.007	0.007	0.010
2014/3/3	0.027	0.035	0.014	0.010	…	0.022	0.002	0.004	0.010
2014/4/1	0.023	0.032	0.021	0.013	…	0.025	-0.007	0.005	0.015
2014/5/1	0.018	0.029	0.017	0.005	…	0.020	0.007	0.005	0.012
2014/6/2	0.022	0.022	0.026	0.005	…	0.021	0.002	0.003	0.013

Annex 3

Covariance matrix

(example: 1 Dec 2005)

Stocks	AXP	BA	CAT	CSCO	…	UTX	VZ	WMT	XOM
AXP	0.0008	0.0002	0.0015	0.0004	…	0.0004	0.0007	0.0000	0.0008
BA	0.0002	0.0015	0.0014	0.0009	…	0.0006	0.0006	-0.0005	0.0017
CAT	0.0015	0.0014	0.0048	0.0013	…	0.0008	0.0013	-0.0001	0.0032
CSCO	0.0004	0.0009	0.0013	0.0025	…	0.0009	0.0007	0.0006	-0.0005
…	…	…	…	…	…	…	…	…	…
UTX	0.0004	0.0006	0.0008	0.0009	…	0.0009	0.0003	-0.0001	-0.0001
VZ	0.0007	0.0006	0.0013	0.0007	…	0.0003	0.0013	0.0001	0.0007
WMT	0.0000	-0.0005	-0.0001	0.0006	…	-0.0001	0.0001	0.0017	-0.0007
XOM	0.0008	0.0017	0.0032	-0.0005	…	-0.0001	0.0007	-0.0007	0.0066

(example: 3 Jan 2006)

Stocks	AXP	BA	CAT	CSCO	…	UTX	VZ	WMT	XOM
AXP	0.0006	-0.0001	0.0011	0.0002	…	0.0002	0.0003	0.0000	0.0008
BA	-0.0001	0.0016	0.0002	0.0003	…	0.0003	-0.0001	-0.0004	0.0012
CAT	0.0011	0.0002	0.0057	0.0018	…	0.0008	0.0014	-0.0002	0.0044
CSCO	0.0002	0.0003	0.0018	0.0028	…	0.0010	0.0007	0.0006	0.0002
…	…	…	…	…	…	…	…	…	…
UTX	0.0002	0.0003	0.0008	0.0010	…	0.0009	0.0002	-0.0001	0.0002
VZ	0.0003	-0.0001	0.0014	0.0007	…	0.0002	0.0010	0.0001	0.0013
WMT	0.0000	-0.0004	-0.0002	0.0006	…	-0.0001	0.0001	0.0017	-0.0007
XOM	0.0008	0.0012	0.0044	0.0002	…	0.0002	0.0013	-0.0007	0.0074

(example: 1 Feb 2006)

Stocks	AXP	BA	CAT	CSCO	…	UTX	VZ	WMT	XOM
AXP	0.0006	0.0000	0.0011	0.0003	…	0.0002	0.0004	0.0000	0.0007
BA	0.0000	0.0014	0.0002	0.0008	…	0.0003	0.0000	-0.0004	-0.0002
CAT	0.0011	0.0002	0.0057	0.0022	…	0.0008	0.0015	-0.0002	0.0037
CSCO	0.0003	0.0008	0.0022	0.0032	…	0.0009	0.0012	0.0005	0.0006
…	…	…	…	…	…	…	…	…	…
UTX	0.0002	0.0003	0.0008	0.0009	…	0.0008	0.0002	-0.0001	0.0006
VZ	0.0004	0.0000	0.0015	0.0012	…	0.0002	0.0013	0.0001	0.0007
WMT	0.0000	-0.0004	-0.0002	0.0005	…	-0.0001	0.0001	0.0017	-0.0006
XOM	0.0007	-0.0002	0.0037	0.0006	…	0.0006	0.0007	-0.0006	0.0036

Annex 4

Weights of portfolio assets calculation (Markowitz model)

(example: 1 Dec 2005)

	A	B	C	D	E	F	G	H	I	J
1	E(x)		Cov Matrix							
2	0.004704		0.0008471	0.000228	0.001476	0.000402	0.000462	0.000341	-0.0003	…
3	0.0278955		0.0002285	0.001531	0.0014	0.000918	0.001332	0.000463	4.29E-05	…
4	0.0181729		0.0014757	0.0014	0.004792	0.001316	0.003094	0.000746	0.000193	…
5	-0.008783		0.0004018	0.000918	0.001316	0.002506	-2.2E-05	0.000239	0.000743	…
6	0.0118244		0.0004621	0.001332	0.003094	-2.2E-05	0.005309	0.001233	0.000101	…
7	…		…	…	…	…	…	…	…	…
8		w EX	w Var	Constrains:						
9	AXP	0	0		(A) Maximum Ex strategy			1. Xi ≥ 0		
10	BA	0	0					2. Σxi = 1		
11	CAT	0	0							
12	CSCO	0	0		(B) Minimum Var strategy			1. Xi ≥ 0		
13	CVX	0	0					2. Σxi = 1		
14	DD	0	0							
15	DIS	0	0.0147198							
16	GE	0	0	Ex	0.029608					
17	GS	0	0.1265438	Var	3.62E-05					
18	HD	0	0.0411743							
19	IBM	0	0							
20	INTC	0	0							
21	JNJ	0	0.425936							
22	JPM	0	0.0421535							
23	KO	0	0							
24	MCD	0	0							
25	MMM	0	0							
26	…	…	…							
27	sum value	1	1							

Weights of portfolio assets – (Maximum Expected return strategy)

Date	AXP	BA	CAT	CSCO	CVX	…	UNH	UTX	VZ	WMT	XOM
2004/12/1	0	0	0	0	0	…	1	0	0	0	0
2005/1/3	0	0	1	0	0	…	0	0	0	0	0
2005/2/1	0	0	1	0	0	…	0	0	0	0	0
2005/3/1	0	0	1	0	0	…	0	0	0	0	0
2005/4/1	0	0	1	0	0	…	0	0	0	0	0
2005/5/2	0	0	1	0	0	…	0	0	0	0	0
…	…	…	…	…	…	…	…	…	…	…	…
2014/1/2	0	1	0	0	0	…	0	0	0	0	0
2014/2/3	0	1	0	0	0	…	0	0	0	0	0
2014/3/3	0	1	0	0	0	…	0	0	0	0	0
2014/4/1	0	1	0	0	0	…	0	0	0	0	0
2014/5/1	0	1	0	0	0	…	0	0	0	0	0
2014/6/2	0	0	0	0	0	…	0	0	0	0	0

Weights of portfolio assets – (Minimum Variance strategy)

Date	AXP	BA	CAT	CSCO	…	UTX	VZ	WMT	XOM
2004/12/1	0.000	0.000	0.000	0.000	…	0.000	0.000	0.000	0.000
2005/1/3	0.000	0.000	0.000	0.000	…	0.000	0.000	0.000	0.000
2005/2/1	0.000	0.000	0.000	0.000	…	0.000	0.000	0.129	0.000
2005/3/1	0.000	0.000	0.000	0.000	…	0.000	0.000	0.000	0.000
2005/4/1	0.000	0.000	0.000	0.000	…	0.000	0.000	0.000	0.000
2005/5/2	0.000	0.000	0.000	0.000	…	0.000	0.000	0.022	0.000
2005/6/1	0.000	0.000	0.000	0.000	…	0.000	0.000	0.000	0.000
2005/7/1	0.098	0.000	0.000	0.000	…	0.232	0.000	0.052	0.000
2005/8/1	0.000	0.000	0.000	0.000	…	0.000	0.000	0.173	0.000
2005/9/1	0.000	0.000	0.000	0.000	…	0.168	0.000	0.044	0.000
2005/10/3	0.000	0.077	0.057	0.000	…	0.178	0.000	0.085	0.000
2005/11/1	0.000	0.095	0.056	0.000	…	0.168	0.000	0.090	0.000
2005/12/1	0.000	0.000	0.172	0.012	…	0.000	0.000	0.107	0.000
2006/1/3	0.000	0.000	0.175	0.012	…	0.000	0.000	0.110	0.000
2006/2/1	0.000	0.000	0.252	0.008	…	0.000	0.000	0.163	0.123
2006/3/1	0.000	0.000	0.267	0.000	…	0.000	0.000	0.046	0.000
2006/4/3	0.000	0.000	0.080	0.000	…	0.070	0.000	0.200	0.160
2006/5/1	0.000	0.000	0.108	0.000	…	0.000	0.000	0.000	0.000
…	…	…	…	…	…	…	…	…	…
2013/1/2	0.000	0.247	0.000	0.000	…	0.000	0.094	0.261	0.000
2013/2/1	0.000	0.220	0.031	0.000	…	0.000	0.124	0.246	0.000
2013/3/1	0.000	0.028	0.171	0.033	…	0.007	0.256	0.254	0.000
2013/4/1	0.000	0.065	0.155	0.001	…	0.147	0.000	0.391	0.000
2013/5/1	0.000	0.144	0.069	0.000	…	0.147	0.064	0.085	0.194
2013/6/3	0.000	0.198	0.139	0.000	…	0.089	0.096	0.000	0.400
2013/7/1	0.000	0.221	0.172	0.030	…	0.000	0.107	0.000	0.390
2013/8/1	0.000	0.349	0.372	0.000	…	0.000	0.175	0.000	0.105
2013/9/3	0.000	0.231	0.326	0.000	…	0.000	0.139	0.000	0.148
2013/10/1	0.000	0.164	0.274	0.018	…	0.000	0.000	0.000	0.000
2013/11/1	0.000	0.159	0.196	0.025	…	0.000	0.000	0.000	0.000
2013/12/2	0.000	0.144	0.159	0.010	…	0.000	0.000	0.000	0.000
2014/1/2	0.000	0.000	0.288	0.083	…	0.014	0.000	0.000	0.000
2014/2/3	0.000	0.000	0.311	0.080	…	0.025	0.000	0.000	0.000
2014/3/3	0.000	0.022	0.356	0.085	…	0.030	0.000	0.000	0.000
2014/4/1	0.000	0.000	0.235	0.122	…	0.033	0.215	0.000	0.000
2014/5/1	0.000	0.000	0.319	0.004	…	0.048	0.230	0.000	0.000
2014/6/2	0.000	0.000	0.279	0.097	…	0.181	0.118	0.000	0.000

Annex 5

Weights of portfolio assets calculation (Black's model)

(example: 1 Dec 2005)

	A	B	C	D	E	F	G	H	I	J
1	E(x)		Cov Matrix							
2	0.004704		0.0008471	0.0002285	0.0014757	0.0004018	0.0004621	0.0003409	-0.0003046	…
3	0.027896		0.0002285	0.0015310	0.0014000	0.0009185	0.0013315	0.0004634	0.0000429	…
4	0.018173		0.0014757	0.0014000	0.0047923	0.0013155	0.0030945	0.0007456	0.0001930	…
5	-0.008783		0.0004018	0.0009185	0.0013155	0.0025060	-0.0000217	0.0002388	0.0007425	…
6	0.011824		0.0004621	0.0013315	0.0030945	-0.0000217	0.0053085	0.0012327	0.0001014	…
7	…		…	…	…	…	…	…	…	…
8		w EX	w Var	abs w Var	Constrains:					
9	AXP	0	0.0749661	0.0749661		(A) Maximum Ex strategy			1. Xi ≥ -1	
10	BA	0	0.0856995	0.0856995					2. Σxi = 1	
11	CAT	0	-0.082269	0.0822695						
12	CSCO	0	-0.018493	0.018493		(B) Minimum Var strategy			1. Xi ≥ -1	
13	CVX	0	0.0869289	0.0869289					2. Σxi = 1	
14	DD	0	-0.065892	0.065892					3. Σ|xi|≤3	
15	DIS	0	0.12042	0.12042						
16	GE	0	0.0475782	0.0475782	Ex	0.039182212				
17	GS	0	0.1070514	0.1070514	Var	3.62955E-15				
18	HD	0	0.0579174	0.0579174						
19	IBM	0	-0.045839	0.045839						
20	INTC	0	-0.092427	0.0924273						
21	JNJ	0	0.1468714	0.1468714						
22	JPM	0	0.0500601	0.0500601						
23	KO	0	0.0698728	0.0698728						
24	MCD	0	-0.042268	0.0422675						
25	MMM	0	0.0161364	0.0161364						
26	…	…	…	…						
27	sum value	1	1	1.7936749						

Weights of portfolio assets – (Maximum Expected return strategy)

Date	AXP	BA	CAT	CSCO	CVX	…	UNH	UTX	VZ	WMT	XOM
2004/12/1	0	0	0	0	0	…	0	-1	0	0	0
2005/1/3	0	0	2	0	0	…	0	0	0	0	0
2005/2/1	0	0	2	0	0	…	0	0	0	0	0
2005/3/1	0	0	2	0	0	…	0	0	0	0	0
2005/4/1	0	0	2	0	0	…	0	0	0	0	0
2005/5/2	0	0	2	0	0	…	0	0	0	0	0
…	…	…	…	…	…	…	…	…	…	…	…
2014/1/2	0	2	0	0	0	…	0	0	0	0	0
2014/2/3	0	2	0	0	0	…	0	0	0	0	0
2014/3/3	0	2	0	0	0	…	0	0	0	0	0
2014/4/1	0	2	0	0	0	…	0	-1	0	0	0
2014/5/1	0	2	0	0	0	…	0	0	0	0	0
2014/6/2	0	0	0	0	0	…	0	0	0	0	0

Weights of portfolio assets – (Minimum Variance strategy)

Date	AXP	BA	CAT	CSCO	…	UTX	VZ	WMT	XOM
2004/12/1	0.075	0.086	-0.082	-0.018	…	0.014	-0.028	0.084	-0.017
2005/1/3	0.277	0.151	-0.102	-0.059	…	-0.023	-0.022	0.031	-0.081
2005/2/1	0.190	0.131	-0.128	-0.057	…	-0.008	-0.032	0.042	-0.039
2005/3/1	0.545	0.100	-0.113	-0.070	…	-0.033	-0.058	0.056	-0.025
2005/4/1	0.607	0.108	-0.118	-0.072	…	-0.034	-0.052	0.057	-0.023
2005/5/2	0.551	0.111	-0.113	-0.067	…	-0.035	-0.042	0.028	-0.030
2005/6/1	0.307	0.045	-0.152	-0.081	…	-0.017	0.039	0.004	-0.058
2005/7/1	0.490	0.067	-0.135	-0.049	…	0.044	0.052	0.043	-0.094
2005/8/1	0.302	0.029	-0.084	-0.007	…	0.051	0.047	0.123	-0.009
2005/9/1	0.075	0.033	0.007	-0.003	…	0.119	0.015	0.122	0.019
2005/10/3	0.053	0.084	0.023	-0.002	…	0.094	0.024	0.126	0.038
2005/11/1	0.060	0.094	0.066	-0.027	…	0.096	-0.010	0.128	0.041
2005/12/1	0.012	0.062	0.101	-0.004	…	0.086	0.032	0.124	0.072
2006/1/3	0.012	0.062	0.101	-0.004	…	0.086	0.032	0.124	0.072
2006/2/1	-0.135	0.027	0.134	-0.016	…	0.065	-0.007	0.159	0.086
2006/3/1	0.005	0.012	0.131	-0.035	…	0.093	-0.025	0.114	0.072
2006/4/3	-0.129	0.024	0.058	-0.019	…	0.122	0.017	0.132	0.091
2006/5/1	-0.359	0.020	0.100	0.005	…	0.056	-0.009	0.120	0.069
…	…	…	…	…	…	…	…	…	…
2013/1/2	-0.083	0.224	-0.012	0.046	…	0.059	0.174	0.264	0.104
2013/2/1	-0.026	0.224	0.095	0.024	…	0.007	0.118	0.237	0.111
2013/3/1	0.000	0.059	0.288	0.046	…	0.055	0.231	0.296	0.127
2013/4/1	0.000	0.086	0.242	0.043	…	0.121	0.159	0.348	0.120
2013/5/1	-0.052	0.187	0.147	0.151	…	0.092	0.175	0.153	0.183
2013/6/3	-0.029	0.178	0.142	0.141	…	0.076	0.177	0.141	0.183
2013/7/1	0.029	0.159	0.152	0.123	…	0.065	0.221	0.036	0.135
2013/8/1	0.004	0.334	0.442	-0.095	…	0.086	0.199	0.000	0.138
2013/9/3	0.178	0.197	0.460	0.000	…	0.136	0.202	0.003	0.104
2013/10/1	0.011	0.070	0.253	0.120	…	0.161	0.116	-0.049	0.019
2013/11/1	0.000	0.079	0.237	0.149	…	0.168	0.117	-0.066	-0.002
2013/12/2	0.000	0.070	0.233	0.143	…	0.171	0.103	-0.058	0.027
2014/1/2	0.012	-0.031	0.254	0.202	…	0.124	0.086	-0.116	-0.056
2014/2/3	-0.037	0.035	0.316	0.178	…	0.079	0.017	-0.072	-0.023
2014/3/3	0.106	0.067	0.286	0.165	…	0.140	0.101	-0.030	-0.018
2014/4/1	0.004	0.080	0.233	0.159	…	0.134	0.167	-0.037	-0.049
2014/5/1	-0.077	0.083	0.260	0.099	…	0.149	0.142	-0.035	-0.023
2014/6/2	0.043	0.031	0.288	0.094	…	0.164	0.134	-0.042	-0.095

Annex 6

The complete records of calculation and dataset is available on additional CD